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Wind Stresses in the Steel Frames of Office Buildings

WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS

BY

CAMILLO WEISS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY CAMILLO WEISS

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WIND STRESSES IN THE STEEL FRAMES OF OFFICE BUILDINGS.

I. Preliminary.

1. Acknowledgment. The investigation herein presented was undertaken upon a suggestion of Mr. W. M. Wilson, Assistant Professor of Structural Engineering at the University of Illinois, as a continuation of his analysis of wind stresses in collaboration with Mr. G. A. Maney, presented in Bulletin No. 80 (1915) of the Engineering Experiment Station at the University of Illinois, entitled "Wind Stresses in the Steel Frames of Office Buildings". The method employed is the approximate slope-deflection method proposed by them. It is based on a simple relation existing between the slope of the elastic curve and the ordinates of the moment diagram of a beam in flexure. The assumptions are stated in another paragraph.

The thesis has been prepared under Professor Wilson's direct supervision. Thanks are due him for valuable advice, helpful suggestions and constructive criticism.

Acknowledgment should be made of the excellent library facilities offered for research by the University of Illinois.

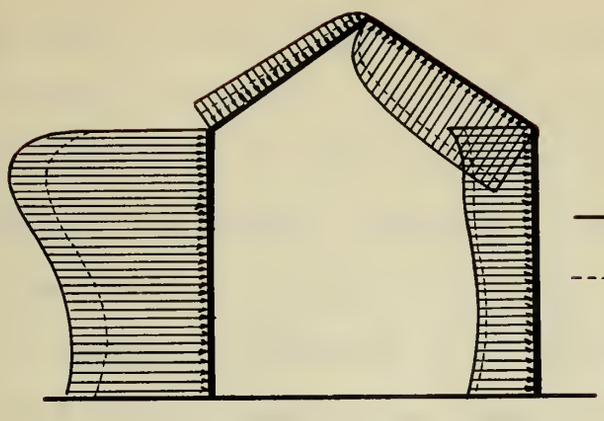
2. Notes on Wind Pressures. "Every engineer, or even every person who ever sees an engineer has a personal interest in the effect of wind on structures. That is the subject of the present book". So writes the editor of Engineering News in the preface to R. Fleming's "Six Monographs on Wind Stresses", which is the most recent (1915) and most comprehensive of the treatises on this subject. As may be seen from the bibliography, there is a comparatively large literature on wind velocity and pressure, but little has

been published about the stresses caused in structures and their calculation. It is generally agreed that the pressure w on a plane surface perpendicular to the direction of the wind varies with the square of its velocity v, or

$$w = k\ v^2$$

If w is expressed in lbs. per sq. ft. and v in miles per hour, k has been found by tests to vary between .003 and .004. The latter figure, as the more conservative, is generally recommended in practice. The coefficient k has to be modified for wind pressure on surfaces inclined towards its direction. Duchemin's formula has been found to agree closely with tests carried out by S. P. Langley in 1888. Values of k have been stated also for the pressure of wind on nonplanar surfaces.

So far one would be led to believe that the wind pressure is a quantity fairly well explored. This, however, is far from being true. Wind is not a current of steadily flowing air as is generally assumed. It moves in shocks and gusts, interspersed with minor waves. It changes velocity and direction not only from one moment to the next, but also with its distance from the ground. The unit pressure on large surfaces appears to be less than that on small ones. There is no certainty as to the point of application and to direction. Tests on small models as well as on test buildings show a suction and up-lifting effect on the leeward side, the distribution and intensity of which depend, amongst other things, upon the conditions of the sides, namely, whether they are entirely or partly closed or entirely open. Fig. 1, taken from "Smith, Wind Pressure on Buildings" shows the distribution of pressure and the effect of openings in walls.



Effect of Openings in Walls.

Fig. 1.

References to tests conducted by Langley, Kernot, Irminger, Stanton, Boardman, Smith and others may be found in the bibliography. Of all the uncertainties the wave action of the wind is probably the most dangerous one. In discussing a paper by C. Shaler Smith on "Wind Pressure upon Bridges" (Transactions, American Society of Civil Engineers, Vol. X, 1880) Mr. Charles A. Smith writes:". . . . It is possible that such action of the wind could produce oscillations in the structure itself, if the periods agreed with that of lateral vibration, or if one was a multiple of the other . . ." In a very instructive report on "Wind Pressures in the St.Louis Tornado, May 27th, 1896" by J. Baier, Professor Langley is quoted as stating, "that the wind is not even an approximately uniform moving mass of air, but consists of a succession of very brief pulsations of varying amplitude, and that relatively to the mean movement of the wind, these are of varying directions." That "these very rapid variations in the pressure take place inside of a second or two of time" has been stated by Professor Marvin, who is further quoted as follows:

"The formula $w = 0.004\ v^2$ gives a mean pressure corresponding to a mean wind velocity. It is important to note that momentary

pressures as much as 35% in excess of the above mean pressure may continually occur and recur. If their rate of occurrence be at all synchronous with a natural time of vibration of the structure or any part thereof, remarkable effects may follow."

Experiments on Chicago buildings under heavy winds show the time of a complete vibration to be about two seconds.

It is little wonder then that accurate calculations of wind stresses have been discouraged by theorists and practitioners alike. "In view of the uncertainty as to the intensity and distribution of wind pressures, refinements in the computation of wind load stresses are unwarranted", says Professor Marburg; and Mr. R. Fleming concludes his chapter on Wind Pressure Formulas and Their Experimental Basis as follows: "Structures have long been designed with satisfactory results to withstand wind pressure. The bracing at times may have been excessive, but in the absence of better knowledge on the subject, engineers can not radically depart from present practice."

3. Purpose of Thesis. When a structure is designed to withstand external forces, three problems have to be solved, each one based upon the preceding one. The first problem is to determine the point of application, the direction and the magnitude of each force on the structure. The second consists in finding the internal forces necessary to balance the external forces. The third problem is to dispose of the building material so as to provide for the internal forces in the safest and most economic way, interfering as little as possible with the purpose of the structure. The three problems should be approached independently.

As concerns wind stresses, the first problem is far from

being solved, while the handling of the third has resulted in a highly developed art. The second problem has reached a stage between the first and the third. It is receiving growing attention. Frames are being investigated as statically indeterminate units and their elastic properties taken into consideration. Papers by Professor Smith of Purdue and Professor Wilson of the University of Illinois treated the problem from this point of view and were well received by the practice.

The ultimate aim of all these elaborate investigations is to devise a method of determining wind stresses in the steel frames of office buildings which will take into account all the important variables and at the same time be simple enough to appeal to the designer.

The purpose of this thesis is to aid in approaching this end.

4. Scope of Thesis. The problem dealt with in this thesis is the determination of the wind stresses in the steel frames of office buildings. Figure 2 represents such a frame with a wind load W acting on its upper portion.

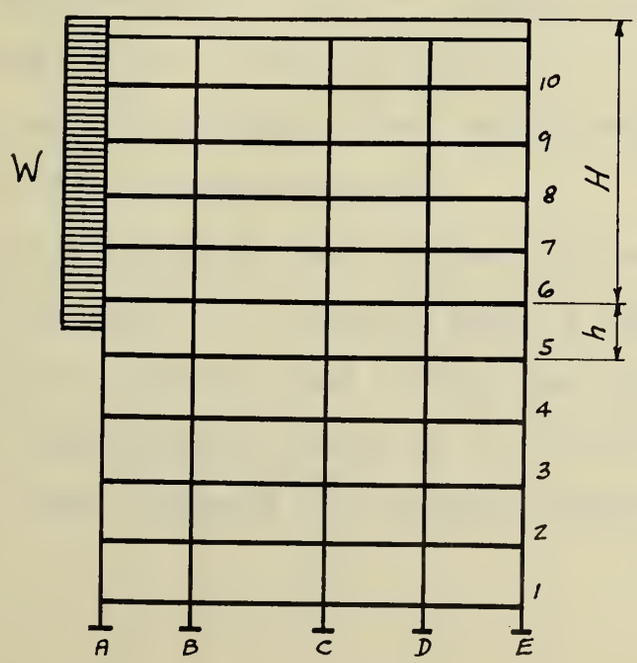


Fig. 2.

The distribution of the horizontal shear in a story among the columns of a bent is a function of the moments of inertia divided by the lengths of the columns and the girders, i. e., the quantity $\frac{I}{\ell} = K$ of the members. Furthermore, the distribution of the horizontal shear is independent of the absolute values of these quantities K, but it is governed by their relative values. It is governed by the ratios of the K of each column to the K of every other column, by the ratios of the K of each column to the K of every girder and by the ratios of the K of each girder to the K of every other girder. The following analysis has been limited to the case where the K's of the columns in the story above the one in which the stresses are to be determined are equal to the K's of the columns of the latter story and where the K's of all girders in a story are equal. That means, in a bent of unequal spans, that the moment of inertia of each girder is proportional to its span length. Where all spans are equal, the moments of inertia of all girders are equal. The quantity K for the columns was made a variable and equal to a fraction or a multiple of the K of the girders; also equal to zero and equal to infinity, thus establishing the extreme limits between which the stresses may vary.

The influence of the K of each column upon the moments has been investigated in a one-span bent, a two-span bent, a three-span bent, a four-span bent and a five-span bent; also the influence of equal and simultaneous changes of the K's of two and three columns. In every instance all possible combinations have been considered.

Likewise the influence of equal simultaneous changes of the K's of the girders has been investigated when the K's of the columns remain constant.

In investigating the effect of the changes in the K's of the members upon the distribution of the stresses, the K's of the members which remain constant are taken equal to unity. The K's of the members which are changed are taken equal to 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, and infinity.

The moments due to wind load at the ends of all columns in a story were determined in each case and expressed in terms of Wh (i.e., wind load times story height). These moments are given in tables and respresented in diagrams. The results have been discussed and conclusions drawn from them.

5. Notation. The following notation has been used throughout this thesis:

A, B, C, etc., = the columns of a bent, beginning at the left and reading toward the right;
= also the coefficients of Wh (the moments at the ends of the corresponding column).

a, b, c, etc., = distance in feet of the corresponding columns from the neutral axis of a bent.

D = direct stress in lbs. in the columns of a given story.

D' = direct stress in lbs. in the columns of the story above.

d = deflection of the columns in a story height.

E = modulus of elasticity.

e = an error.

H = height in feet of a building above the top of a given story.

h = length of a column measured between neutral axes of the girders.

h = height [in feet] of the story above.

I = moment of inertia

ℓ = length of a girder measured between neutral axes of the columns.

$K = \frac{I}{\ell}$ for girders and $\frac{I}{h}$ for columns.

M = bending moment.

m = number of a given story.

$N = a^2 + b^2 + c^2 + \text{etc.}$

n = total number of stories in a given bent.

P = average wind load in lbs. in a story height.

$P = W - W'$.

$R = \frac{d}{h}$

r = number of any column counting from left to right.

t = total number of columns in a story.

v = velocity in miles per hour.

W = total wind load in lbs. above mid-height of a given story.

W' = corresponding wind load in the story above.

w = wind pressure in lbs. per sq. ft.

θ = change in the slope of the tangent to the elastic curve.

Σ = a sum.

Subscripts are added to the letters to indicate the particular part of a bent to which a given symbol applies.

Method I = Mr. Robins Fleming's method I as described in his "Six Monographs on Wind Stresses". *)

Method IV = Prof. Albert Smith's method as described under II-A in R. Fleming's "Six Monographs on Wind Stresses." **)

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*)For assumptions see p. 77.

**)For assumptions see p. 74.

II. Theory Upon which the Investigation is Based.

6. Outline of the Analysis Used.*)

"It can be proven that the moment at an end of a member of a frame is a function of the changes in the slopes of the tangents to the elastic curve of the member at its ends and of the deflection of one end of the member relative to the other end:

$$M_{AB} = 2EK_{AB}(2\,\theta_A + \theta_B - 3R)$$

"In the strained position, all the columns and girders which intersect at one point have been subjected to the same change in slope (see assumption 1); the vertical deflections of the ends of all girders are equal to zero; and the horizontal deflections of the tops of all columns of a story are equal (see assumption 2).

"Consider any story of a bent. Take the point of intersection of the neutral axes of a column and a girder as a free body. It is in equilibrium under the action of the moments at the extremities of the columns and girders which intersect at the point. Each of the moments may be expressed in terms of the changes in the slopes at the extremities of the member, and the deflection of one end of the member relative to the other. A moment equation can therefore be written for each point where the columns and girders intersect, and the only unknown quantities will be the changes in the slopes at the extremities of the columns and the horizontal deflections of the columns in a story.

"If all the columns of a story be taken together as a free body, the sum of the moments at the two extremities of all the columns will be balanced by a couple whose moment is equal to the total shear on the story multiplied by the story height. The shear and the height of the story are known, and the moments in the

*) Quoted from W. M. Wilson and G. A. Maney "Wind Stresses In The Steel Frames of Office Buildings", Bulletin No. 80, Engineering Experiment Station, U. of I.

columns can be expressed in terms of the slopes and the deflections at their extremities the same as in the previous equations. It is therefore possible to write as many equations for each story as there are columns in the story, plus one. As the only unknown quantities in these equations are the changes in the slopes at the extremities of the columns and the deflection in a story common to all columns, there are as many equations per story as there are unknowns. By solving these equations the slopes and the deflections can be determined. Knowing the slopes and the deflections, the moments can be computed.

"The product of the shear on a member and the length of the member is equal to the algebraic sum of the moments at the extremities of the member. Since the moments and the lengths of the member are known the shear can be computed.

"With the shears in the girders known, the direct stress in any column can be determined by taking the column as a free body and equating the sum of the vertical forces to zero."

The slope-deflection method as outlined is based upon assumptions 1 to 4 inclusive, of article 7. The slope-deflection method is further modified by assumptions 5 and 6. As modified, it is designated in Bulletin No. 80, Engineering Experiment Station, University of Illinois, as the approximate slope-deflection method. The approximate slope-deflection method is used in this analysis. It is to be noted that if assumptions 5 and 6 were true, the approximate slope-deflection and the slope-deflection method would furnish identical results.

7. Assumptions Upon Which the Analyses are Based. The analyses are based upon the following assumptions:

(1) The connections between the columns and girders are perfectly rigid.

(2) The change in the length of a member due to the direct stress is equal to zero.

(3) The length of a girder is the distance between the neutral axes of the columns which it connects, and the length of a column is the distance between the neutral axes of the girders which it connects.

(4) The deflection of a member due to the internal shearing stresses is equal to zero.

These four assumptions suffice to furnish as many equations as there are unknowns, provided the entire frame is treated as one unit. This means a total number of simultaneous equations equal to the total number of joints plus the total number of stories in the frame. The equations for a single story can not be solved independently of the others. In order to isolate the equations of the individual stories the following assumptions are added:

(5) The changes in the slope at the top of a column in the story above and in the story below the one in which the stresses are to be determined, are equal to the change in the slope at the top of the corresponding column in the latter story. This is equivalent to saying the point of contraflexure of the column under consideration is at its midheight.

(6) The ratio of the deflection to the length of the columns in the story above the one in which the stresses are to be determined, is equal to the ratio of the deflection to the length of the columns in the latter story.

(7) Shears and story heights in two consecutive stories are equal.

Under these conditions the general equations for the four-span bent, for instance, would read:

$$\frac{\theta_A}{R}(6K_A + 2K) + \frac{\theta_B}{R} \cdot K = 6K_A \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$\frac{\theta_A}{R} \cdot K + \frac{\theta_B}{R}(6K_B + 4K) + \frac{\theta_C}{R} \cdot K = 6K_B \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{\theta_B}{R} \cdot K + \frac{\theta_C}{R}(6K_C + 4K) + \frac{\theta_D}{R} \cdot K = 6K_C \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$\frac{\theta_C}{R} \cdot K + \frac{\theta_D}{R}(6K_D + 4K) + \frac{\theta_E}{R} \cdot K = 6K_D \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$\frac{\theta_D}{R} \cdot K + \frac{\theta_E}{R}(6K_E + 2K) = 6K_E \quad \ldots\ldots\ldots\ldots \quad (5)$$

8. Notes on the Calculation of Moments in Columns and Girders. (a) General Case. The application of the principles outlined in the preceding articles will be demonstrated in detail for the four-span bent Figure 3. The moment at the end A of a member AB

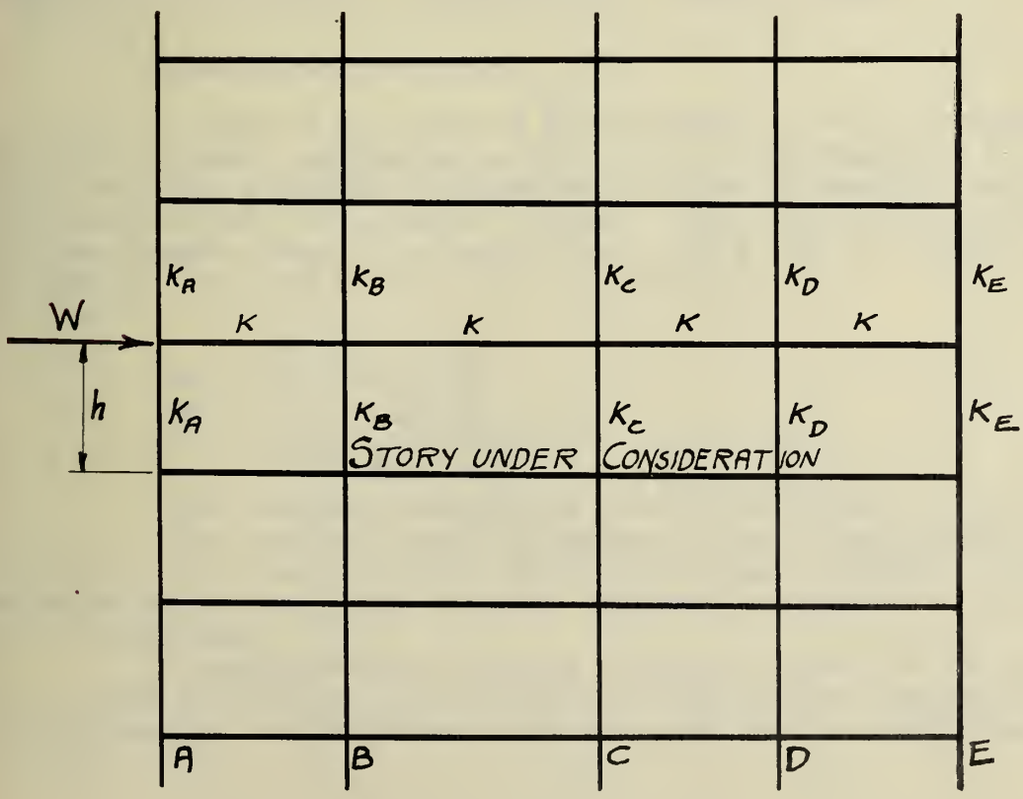


Fig. 3.

under flexure due to external couples at the ends

$$M_{AB} = 2EK_{AB}(2\theta_A + \theta_B - 3R)$$

For each joint under equilibrium the algebraic sum of the moments at the ends meeting there equals zero.

For joint A,

$$M_A + M_A + M_{AB} = 0,$$

or

$$2EK_A(2\theta_A + \theta_A - 3R) + 2EK_A(2\theta_A + \theta_A - 3R) + 2EK(2\theta_A + \theta_B) = 0$$

Dividing both sides of the equation by 2E and reducing,

$$\theta_A(6K_A + 2K) + \theta_B.K = R.6K_A$$

Similarly the equations for joints B, C, D and E are obtained.

Instead of using θ's and R's as unknown quantities, the ratio of the former to the latter has been introduced. The advantage thus gained is to eliminate one unknown and therewith one equation. This end may be obtained in every case where there is but one R and but one equation containing a constant term. *)

---

*) Then, the ratios $\frac{\theta}{R}$ are constant values peculiar to the given frame, depending solely on its dimensions, i.e., the values of K, and are entirely independent of the direction, magnitude, and distribution of the external loads. This condition exists always for the frames under consideration, and a set of equations of the following form can be written for every bent:

$$\begin{aligned} ax + by &= k_1 \\ bx + cy + dz &= k_2 \\ dy + ez + fu &= k_3 \\ \text{----------} \\ pw + qt &= k_r \end{aligned}$$

The unknown quantities are x, y, z, u ..... w, t.

The time required to solve these simultaneous equations increases very rapidly with each additional unknown. If a bent contains more than four or five columns, a graphical solution may be of advantage.

A comparatively simple method is said to be contained in a paper by Massau, published in 1887 in the "Annales de l'Association des Ingénieurs Sortis des Écoles Spéciales de Gand" (Ghent).

Dividing both sides of the above equation by R, furnishes

$$\frac{\theta_A}{R}(6K_A + 2K) + \frac{\theta_B}{R} \cdot K = 6K_A \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Similarly for joint B,

$$\frac{\theta_A}{R} \cdot K + \frac{\theta_B}{R}(6K_B + 4K) + \frac{\theta_C}{R} \cdot K = 6K_B \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

For joint C,

$$\frac{\theta_B}{R} \cdot K + \frac{\theta_C}{R}(6K_C + 4K) + \frac{\theta_D}{R} \cdot K = 6K_C \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

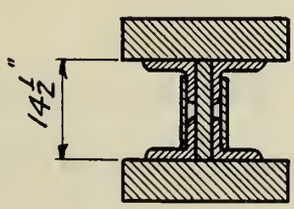
For joint D,

$$\frac{\theta_C}{R} \cdot K + \frac{\theta_D}{R}(6K_D + 4K) + \frac{\theta_E}{R} \cdot K = 6K_D \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

For joint E,

$$\frac{\theta_D}{R} \cdot K + \frac{\theta_E}{R}(6K_E + 2K) \qquad = 6K_E \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

The absolute values of K may vary considerably. Take an 8" I-beam, 14'-0" long, with a moment of inertia I = 56.9 in.^{4} $\frac{I}{\ell} = K = \frac{56.9}{14 \times 12} = .34$ in.^{3}, practically $\frac{1}{3}$ in.^{3} On the other hand for a column of the lower tier in the Equitable Building made up as shown in Fig. 4, with a moment of inertia I = 26061 in.^{4} and a length



Moment of Inertia I:

4 Ls 8"x6"x1"	1786.	in.^{4}
Web 14"x½"	457.4	in.^{4}
2-Cov. 24"x6"	23563.2	in.^{4}
2-Side Pls. 11½"x1"	254.	in.^{4}
Total I =	26061.	in.^{4}

Fig. 4.

Column Section of Equitable Building.

of 11'-0", $\frac{I}{h} = \frac{26061}{11 \times 12} = K = 198$ in.^{3}, practically 200 in.^{3} The K of the large girders in this building is about 80 in.^{3}

However, as the distribution of the horizontal shear is not governed by the absolute values of the quantity K but by their relative values, the K's of the girders are introduced as unity, and the K's of the columns as 0, 1/2, 1, 3/2, 2 and ∞ times unity.

(b) All K's Finite Quantities. Consider, for instance, the case in which the K's of the girders are equal to unity and the K's of columns A, B and C are equal to unity and the K's of columns D and E are equal to one-half the K's of columns A, B and C. In symbols,

$$K_A = K_B = K_C = 1, \qquad K = 1, \qquad K_D = K_E = \frac{1}{2}.$$

Introducing these values into equations (1) to (5)

$$8\frac{\theta_A}{R} + \frac{\theta_B}{R} = 6 \quad \ldots\ldots\ldots\ldots \quad (1')$$

$$\frac{\theta_A}{R} + 10\frac{\theta_B}{R} + \frac{\theta_C}{R} = 6 \quad \ldots\ldots\ldots\ldots \quad (2')$$

$$\frac{\theta_B}{R} + 10\frac{\theta_C}{R} + \frac{\theta_D}{R} = 6 \quad \ldots\ldots\ldots\ldots \quad (3')$$

$$\frac{\theta_C}{R} + 7\frac{\theta_D}{R} + \frac{\theta_E}{R} = 3 \quad \ldots\ldots\ldots\ldots \quad (4')$$

$$\frac{\theta_D}{R} + 5\frac{\theta_E}{R} = 3 \quad \ldots\ldots\ldots\ldots \quad (5')$$

The method of solving these equations used in this thesis is that of elimination as demonstrated in the following table:

Elimination of Unknowns from the Equations for a Four-Span Bent

No.	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{\theta_E}{R}$	R. H.
(1')	8	1				6
(2')	1	10	1			6
(a)	8	80	8			48
(a)-(1')		79	8			42
(3')		1	10	1		6
(b)		79	790	79		474
(b)-(3')			782	79		432
(4')			1	7	1	3
(c)			782	5474	782	2346
(c)-(4')				5395	782	1914
(5')				1	5	3
(d)				5395	26975	16185
(d)-(5')					26193	14271

$$\frac{\theta_E}{R} = \frac{14271}{26193} = .544 \qquad 1 - \frac{\theta_E}{R} = .456$$

$$\frac{\theta_D}{R} = \frac{7224}{26193} = .276 \qquad 1 - \frac{\theta_D}{R} = .724$$

$$\frac{\theta_C}{R} = \frac{13740}{26193} = .525 \qquad 1 - \frac{\theta_C}{R} = .475$$

$$\frac{\theta_B}{R} = \frac{12534}{26193} = .478 \qquad 1 - \frac{\theta_B}{R} = .522$$

$$\frac{\theta_A}{R} = \frac{18078}{26193} = .690 \qquad 1 - \frac{\theta_A}{R} = .310$$

The moment at the end of column A

$$M_A = 2EK_A(2\theta_A + \theta_A - 3R) = 6EK_A(\theta_A - R)$$

or,

$$M_A = -\ 6ER.K_A.(1 - \frac{\theta_A}{R})$$

In terms of Wh

$$M_A = -A.Wh$$

Hence

$$A = \frac{6ER}{Wh}.K_A.(1 - \frac{\theta_A}{R}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (I)$$

Similarly

$$B = \frac{6ER}{Wh}.K_B.(1 - \frac{\theta_B}{R}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (II)$$

$$C = \frac{6ER}{Wh}.K_C\ (1 - \frac{\theta_C}{R}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (III)$$

$$D = \frac{6ER}{Wh}.K_D\ (1 - \frac{\theta_D}{R}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (IV)$$

$$E = \frac{6ER}{Wh}.K_E\ (1 - \frac{\theta_E}{R}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (V)$$

The quantity $\frac{6ER}{Wh}$ may be determined from the condition that the sum of the moments at the ends of the columns in a story equals the product of the shear in that story times the story height.

$$2M_A + 2M_B + 2M_C + 2M_D + 2M_E = -\ Wh$$

$$6ER\left[2K_A(1 - \frac{\theta_A}{R}) + 2K_B(1-\frac{\theta_B}{R}) + 2K_C(1-\frac{\theta_C}{R}) + 2K_D(1-\frac{\theta_D}{R}) + 2K_E(1-\frac{\theta_E}{R})\right] = Wh$$

hence,

$$\frac{6ER}{Wh} = \frac{1}{2K_A(1-\frac{\theta_A}{R}) + 2K_B(1-\frac{\theta_B}{R}) + 2K_C(1-\frac{\theta_C}{R}) + 2K_D(1-\frac{\theta_D}{R}) + 2K_E(1-\frac{\theta_E}{R})}.(6)$$

Applied to the above example,

$$\frac{6ER}{Wh} = \frac{1}{(2 \times .310) + (2 \times .522) + (2 \times .475) + (1 \times .724) + (1 \times .456)}$$

$$\frac{6ER}{WH} = \frac{1}{3.794} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6')$$

Introducing this quantity into Equations (I) to (V), the expressions for the coefficients of Wh,

$A = \frac{1 \times .310}{3.794} = .082$	$M_A = -.082\ Wh$
$B = \frac{1 \times .522}{3.794} = .138$	$M_B = -.138\ Wh$
$C = \frac{1 \times .475}{3.794} = .125$	$M_C = -.125\ Wh$
$D = \frac{\frac{1}{2} \times .724}{3.794} = .095$	$M_D = -.095\ Wh$
$E = \frac{\frac{1}{2} \times .456}{3.794} = .060$	$M_E = -.060\ Wh$
$A + B + C + D + E = .500$	$M_A + M_B + M_C + M_D + M_E = -.500\ Wh$

The results are the moments at the ends of the columns. They are expressed as coefficients of Wh. The results have been presented in tables and in two sets of diagrams. The values of the $\frac{\theta}{R}$'s and the reciprocal values of the quantities $\frac{6ER}{Wh}$ have also been tabulated.

As all girders have equal K's the moment at the end of a girder is equal to the moment in the column at that point if it is an interior column, and twice that, if it is an exterior column.

<u>(c) Some K's zero or infinity.</u> The solution of the equations for finite quantities of K requires no comment. However, if K's of the girders are zero, the solution given on page 16 gives indeterminate values for the M's. For example, in equations (1) to (5), if K = 0

$$\text{all } \frac{\theta}{R}\text{'s} = 1$$

Hence

$$1 - \frac{\theta}{R} = 0,$$

and the coefficients A, B, C, and D of Wh, which have the general form

$$C = K_C \cdot \frac{1 - \frac{\theta_C}{R}}{2\sum(1 - \frac{\theta}{R})} = K_C \cdot \frac{0}{0}$$

are indeterminate quantities.

A similar thing happens, where K for one or more columns becomes ∞. At the end of such a column

$$\frac{\theta}{R} = 1$$

and

$$1 - \frac{\theta}{R} = 0$$

The expression for the coefficient of Wh for column C when K of column C is infinity is

$$C = \frac{K_C \cdot (1 - \frac{\theta_C}{R})}{2\sum(1 - \frac{\theta}{R})}$$

in which $(1 - \frac{\theta_C}{R})$ is for column C and therefore equals zero, and in which $\sum(1 - \frac{\theta}{R})$ is for all the columns and equals a finite quantity. Let this finite quantity be represented by $\frac{1}{q}$.

Then for a column C whose $K_C = \infty$

$$C = q \cdot \infty \cdot 0$$

an indeterminate quantity.

Two examples are given to demonstrate how this difficulty has been avoided.

They are taken from Cases (I) and (II) of the Four-Span Bent represented by Figure 3A.

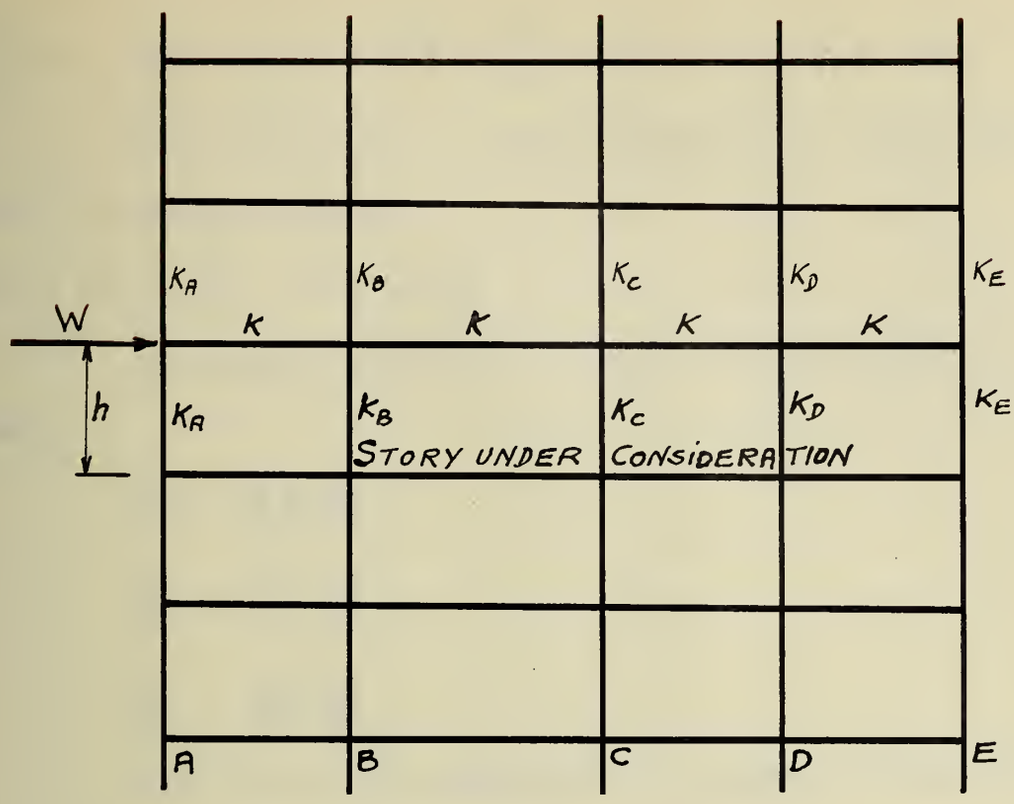


Fig. 3A.

First Example. Case I.

(I). $K_A = K_B = K_C = K_D = K_E = 1$ $\qquad K = 0$

As far as the quantities K are concerned, this bent is symmetrical, and

$$\theta_A = \theta_E$$

and

$$\theta_B = \theta_D$$

The general equations will then read

$$\frac{\theta_A}{R}(6 + 2K) + \frac{\theta_B}{R} \cdot K = 6 \quad \ldots\ldots\ldots\ldots \quad (7)$$

$$\frac{\theta_A}{R} \cdot K + \frac{\theta_B}{R}(6 + 4K) + \frac{\theta_C}{R} \cdot K = 6 \quad \ldots\ldots\ldots\ldots \quad (8)$$

$$\frac{\theta_B}{R} \cdot 2K + \frac{\theta_C}{R}(6 + 4K) = 6 \quad \ldots\ldots\ldots\ldots \quad (9)$$

Eliminating $\frac{\theta_A}{R}$ from equations (7) and (8), we obtain

$$\frac{\theta_B}{R}\left[(6 + 4K)(6 + 2K) - K^2\right] + \frac{\theta_C}{R} \cdot K(6 + 2K) = 6(6 + K) \quad . \; . \; . \; . \; (10)$$

Neglecting K^2 as a small quantity of higher order, equation (10) may be written

$$\frac{\theta_B}{R}(6 + 6K) + \frac{\theta_C}{R} \cdot K = 6 + K \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (10')$$

Combining equations (9) and (10') and neglecting K^2, we obtain

$$\frac{\theta_C}{R} = \frac{3 + 2K}{3 + 5K}$$

$$\frac{\theta_B}{R} = \frac{3 + 5K}{3 + 8K}$$

$$\frac{\theta_A}{R} = \frac{2 + 5K}{2 + 6K}$$

Reducing these quantities to a common denominator, they become

$$\frac{\theta_C}{R} = \frac{6 + 38K}{6 + 44K}$$

$$\frac{\theta_B}{R} = \frac{6 + 38K}{6 + 44K}$$

$$\frac{\theta_A}{R} = \frac{6 + 41K}{6 + 44K}$$

As will be noticed,

$$\frac{\theta_B}{R} = \frac{\theta_C}{R},$$

and

$$2\frac{\theta_A}{R} - \frac{\theta_B}{R} = 1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (11)$$

Equation (11) holds also for all the other bents investigated. For the Five-Span Bent, for instance,

$$\frac{\theta_B}{R} = \frac{\theta_C}{R} = \frac{\theta_D}{R} = \frac{\theta_E}{R} = \frac{2 + 6K}{2 + 8K}$$

and

$$\frac{\theta_A}{R} = \frac{\theta_F}{R} = \frac{2 + 7K}{2 + 8K}$$

hence

$$2\frac{\theta_A}{R} - \frac{\theta_B}{R} = 1$$

or

$$2\theta_A - \theta_B = R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

This relation may be expressed as follows:

If the K's of all columns in a story are equal and finite quantities, and the K's of all girders in the story are equal to zero, the slope-deflections at the ends of all interior columns in this story are equal and the slope-deflections at the ends of the exterior columns are equal. The difference of twice the slope-deflection of an exterior column minus the slope-deflection of an interior column is equal to the ratio R of the horizontal deflection to the height of the story under consideration.

Equation (11) may be written

$$\frac{1 - \frac{\theta_B}{R}}{1 - \frac{\theta_A}{R}} = 2$$

Introducing this relation into the expression for $\frac{6ER}{Wh}$, we obtain for the Four-Span Bent

$$\frac{6ER}{Wh} = \frac{1}{16(1 - \frac{\theta_A}{R})} = \frac{1}{8(1 - \frac{\theta_B}{R})}$$

The coefficients of Wh are

$$A = F = \frac{6ER}{Wh} \cdot (1 - \frac{\theta_A}{R}) = \frac{1 - \frac{\theta_A}{R}}{16(1 - \frac{\theta_A}{R})} = \frac{1}{16}$$

$$B = C = D = E = \frac{6ER}{WH} \cdot (1 - \frac{\theta_B}{R}) = \frac{1 - \frac{\theta_B}{R}}{8(1 - \frac{\theta_B}{R})} = \frac{1}{8}$$

The coefficients of Wh for the interior columns are twice those for the exterior columns.

For the Five-Span Bent we have

$$\frac{6ER}{Wh} = \frac{1}{4(1 - \frac{\theta_A}{R}) + 8(1 - \frac{\theta_B}{R})} = \frac{1}{20(1 - \frac{\theta_A}{R})} = \frac{1}{10(1 - \frac{\theta_B}{R})}$$

The coefficients of Wh are

$$A = F = \frac{6ER}{Wh}(1 - \frac{\theta_A}{R}) = \frac{1 - \frac{\theta_A}{R}}{20(1 - \frac{\theta_A}{R})} = \frac{1}{20}$$

$$B = C = D = E = \frac{6ER}{Wh}(1 - \frac{\theta_B}{R}) = \frac{1 - \frac{\theta_B}{R}}{10(1 - \frac{\theta_B}{R})} = \frac{1}{10}$$

Again, the coefficients for the interior columns are twice those for the exterior columns. In general this relation may be expressed as follows:

If the K's of all columns in a story are equal and finite quantities, and the K's of all girders in the story are equal to zero, the coefficients of Wh for the interior columns are twice those for the exterior columns.

Reference to this relation will be taken in the discussion of "Method IV".

<u>Second Example. Case II.</u>

(II) $K_A = K_B = K_C = K_D = K = 1 \qquad K_E = \infty$

The general equations for this case are:

$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{\theta_E}{R}$	Right Hand of Equation	Equation No.
8	1				6	1"
1	10	1			6	2"
	1	10	1		6	3"
		1	10	1	6	4"
			1	$6K_E + 2$	$6K_E$	5"

Dividing both sides of equation (5") by $K_E = \infty$, we obtain

$$\frac{\theta_E}{R} = 1$$

$\frac{\theta_A}{R}$, $\frac{\theta_B}{R}$, $\frac{\theta_C}{R}$, and $\frac{\theta_D}{R}$ are finite quantities smaller than unity; they are

$$\frac{\theta_A}{R} = .690$$

$$\frac{\theta_B}{R} = .480$$

$$\frac{\theta_C}{R} = .507$$

$$\frac{\theta_D}{R} = .449$$

Substituting in equation (5") the value for $\frac{\theta_D}{R}$, we have

$$.449 + \frac{\theta_E}{R}(6K_E + 2) = 6K_E \quad \ldots\ldots\ldots\ldots\ldots \quad (5a)$$

or

$$6K_E(1 - \frac{\theta_E}{R}) = .449 + 2\frac{\theta_E}{R} \quad \ldots\ldots\ldots\ldots\ldots \quad (5b)$$

Substituting the value of $\frac{\theta_E}{R}$ in the right hand side of equation (5b)

$$\lim. 6K_E(1 - \frac{\theta_E}{R}) = 2.449$$

and

$$\lim. 2K_E(1 - \frac{\theta_E}{R}) = .8163 \quad \ldots\ldots\ldots\ldots\ldots \quad (5c)$$

This value may now be used to determine $\frac{6ER}{Wh}$

$$\frac{6ER}{Wh} = \frac{1}{2\left[(1 - \frac{\theta_A}{R})+(1 - \frac{\theta_B}{R})+(1 - \frac{\theta_C}{R})+(1 - \frac{\theta_D}{R})\right]+2K_E(1 - \frac{\theta_E}{R})} \quad \ldots (6)$$

$$\frac{6ER}{Wh} = \frac{1}{2\left[.310 + .520 + .493 + .551\right] + .816_3} \quad \ldots (6a)$$

$$\frac{6ER}{Wh} = \frac{1}{4.564} \quad \ldots (6b)$$

The coefficients of Wh are:

$$A = \frac{6ER}{Wh} \cdot (1 - \frac{\theta_A}{R}) = \frac{.310}{4.564} = .068$$

$$B = \frac{6ER}{Wh} \cdot (1 - \frac{\theta_B}{R}) = \frac{.520}{4.564} = .114$$

$$C = \frac{6ER}{Wh} \cdot (1 - \frac{\theta_C}{R}) = \frac{.493}{4.564} = .108$$

$$D = \frac{6ER}{Wh} \quad (1 - \frac{\theta_D}{R}) = \frac{.551}{4.564} = .121$$

$$E = \frac{6ER}{Wh} \cdot K_E(1 - \frac{\theta_E}{R}) = \frac{.408_2}{4.564} = .089$$

A check on the equation for $\frac{6ER}{Wh}$ may be obtained in every case by adding all coefficients of Wh. Their sum must always equal five-tenths. That is, A + B + C + D + E = .500.

III. Determination of Moments in Bents of Office Building Frames.

9. One-Span Bent. Figure 5 represents a story of a one-span bent. The $\frac{I}{\ell}$'s of the girders are represented by K, the $\frac{I}{h}$ of Column A is represented by K_A and of Column B by K_B. The shear upon the story is W, and the sum of the moments at the tops and bottoms of both columns is Wh. Then

$$M_A = - A\ Wh$$

$$M_B = - B\ Wh$$

in which A and B are coefficients to be determined.

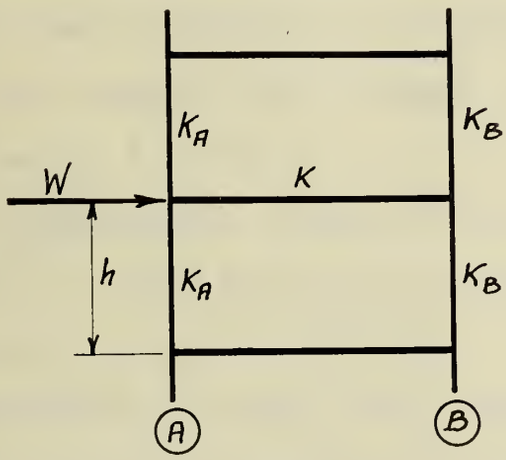


Fig. 5.

The general equations for this bent may be written:

$$\frac{\theta_A}{R}(6K_A + 2K) + \frac{\theta_B}{R} \cdot K = 6K_A \quad . \ . \ . \ (1)$$

$$\frac{\theta_A}{R} \cdot K + \frac{\theta_B}{R}(6K_B + 2K) = 6K_B \quad . \ . \ . \ (2)$$

$\frac{\theta_A}{R}$ and $\frac{\theta_B}{R}$ can be determined from these equations as illustrated on page 16.

$$\frac{6ER}{Wh} = \frac{1}{2K_A(1 - \frac{\theta_A}{R}) + 2K_B(1 - \frac{\theta_B}{R})} \quad . \ . \ . \ (3)$$

As determined on page 17, the coefficients of Wh are:

$$A = \frac{6ER}{Wh} K_A(1 - \frac{\theta_A}{R}) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (4)$$

$$B = \frac{6ER}{Wh} K_B \left(1 - \frac{\theta_B}{R}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (5)$$

The cases investigated are:

(I) $K_A = K_B = 1$

$K = 0,\ 1/2,\ 1,\ 3/2,\ 2,$ and ∞.

(II) $K_A = K = 1$

$K_B = 0,\ 1/2,\ 1,\ 3/2,\ 2$ and ∞.

The values of $\frac{\theta_A}{R}$, $\frac{\theta_B}{R}$, and $\frac{Wh}{6ER}$ are given in Table I, and the moments in the columns in terms of Wh are given in Table II. The change in moments in the columns due to changes in the K's of the girder are shown graphically in Diagram 1, and changes in the moments in the columns due to changes in the K's of the columns are given in Diagrams 2 and 3.

<u>10. Two-Span Bent.</u> Figure 6 represents a story of a two-span bent. The $\frac{I}{\ell}$'s of the girders are represented by K, the $\frac{I}{h}$ of Column A is represented by K_A, of column B by K_B and of column C by K_C. The shear upon the story is W, and the sum of the moments at the tops and bottoms of all three columns is Wh.

Then

$$M_A = -\ A.Wh$$

$$M_B = -\ B.Wh$$

$$M_C = -\ C.Wh$$

in which A, B and C are coefficients to be determined.

TABLE I.

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the One-Span Bent.

K_A	K_B	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{Wh}{6ER}$
1	1	0	1.000	1.000	0
1	1	1/2	.800	.800	.800
1	1	1	.667	.667	1.333
1	1	3/2	.571	.571	1.716
1	1	2	.500	.500	2.000
1	1	∞	0	0	4.000
1	0	1	.800	- .400	.400
1	1/2	1	.692	.462	1.154
1	1	1	.667	.667	1.333
1	3/2	1	.655	.758	1.416
1	2	1	.649	.811	1.458
1	∞	1	.625	1.000	1.625

TABLE II

Coefficients of Wh for the One-Span Bent.

K_A	K_B	K	A	B
1	1	0	.250	.250
1	1	1/2	.250	.250
1	1	1	.250	.250
1	1	3/2	.250	.250
1	1	2	.250	.250
1	1	∞	.250	.250
1	0	1	.500	0
1	1/2	1	.267	.233
1	1	1	.250	.250
1	3/2	1	.244	.256
1	2	1	.241	.259
1	∞	1	.231	.269

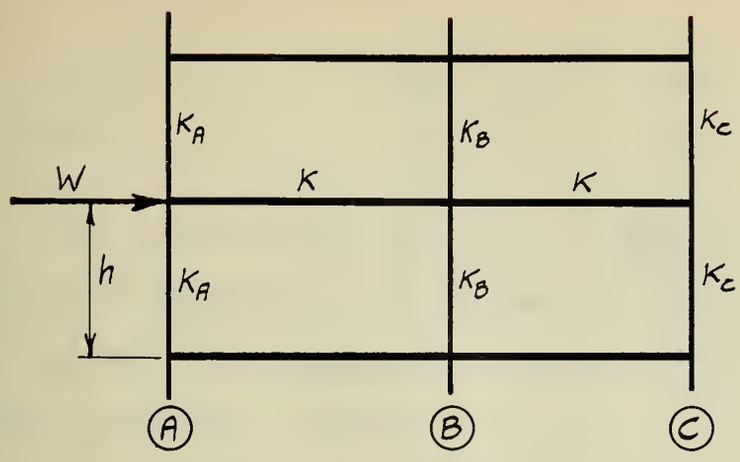


Fig. 6.

The general equations for this bent are:

$$\frac{\theta_A}{R}(6K_A + 2K) + \frac{\theta_B}{R}.K = 6K_A \quad . \; . \; . \; . \; . \quad (1)$$

$$\frac{\theta_A}{R}.K + \frac{\theta_B}{R}(6K_B + 4K) + \frac{\theta_C}{R}\;K = 6K_B \quad . \; . \; . \; . \; . \quad (2)$$

$$\frac{\theta_B}{R}.K + \frac{\theta_C}{R}(6K_C + 2K) = 6K_C \quad . \; . \; . \; . \; . \quad (3)$$

$\frac{\theta_A}{R}$, $\frac{\theta_B}{R}$ and $\frac{\theta_C}{R}$ can be determined from these equations as illustrated on page 16.

$$\frac{6ER}{Wh} = \frac{1}{2K_A(1 - \frac{\theta_A}{R}) + 2K_B(1 - \frac{\theta_B}{R}) + 2K_C(1 - \frac{\theta_C}{R})} \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

The coefficients of Wh are:

$$A = \frac{6ER}{Wh}.K_A(1 - \frac{\theta_A}{R})$$

$$B = \frac{6ER}{Wh}.K_B(1 - \frac{\theta_B}{R})$$

$$C = \frac{6ER}{Wh}.K_C(1 - \frac{\theta_C}{R})$$

The cases investigated are:

(I) $K_A = K_B = K_C = 1$, $K = 0, 1/2, 1, 3/2, 2$ and ∞.

(II) $K_A = K_B = K = 1$, $K_C = 0, 1/2$, etc.

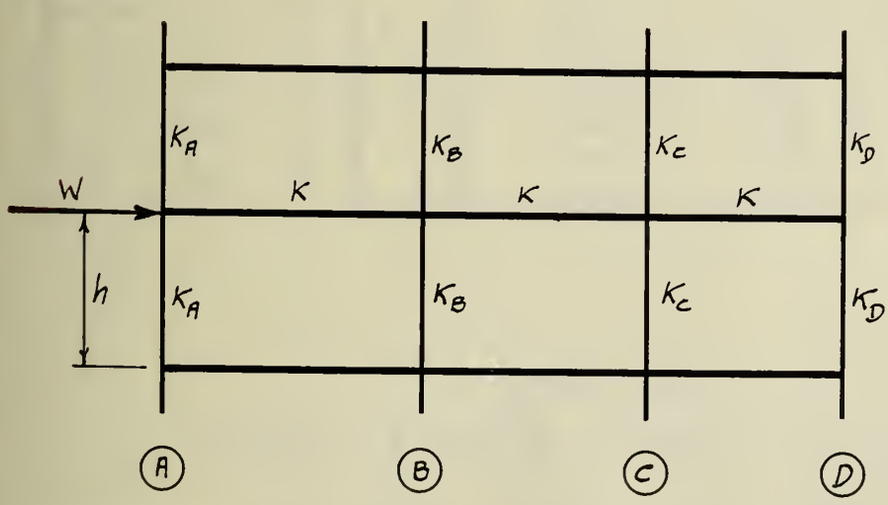
(III) $K_A = K_C = K = 1$, $K_B = 0, 1/2$, etc.

(IV) $K_B = K = 1$, $K_A = K_C = 0,\ 1/2,\ 1,\ 3/2,\ 2,$ and ∞.

(V) $K_A = K = 1$, $K_B = K_C = 0,\ 1/2,$ etc.

The values of $\frac{\theta_A}{R}$, $\frac{\theta_B}{R}$, $\frac{\theta_C}{R}$ and $\frac{Wh}{6ER}$ are given in Tables III and IV, and the moments in the columns in terms of Wh are given in Tables V and VI. The changes in moments in the columns due to equal and simultaneous changes in the K's of the girders are shown graphically in Diagram 4, and changes in the moments in the columns due to changes in the K's of the columns are given in Diagrams 5 to 16 inclusive.

11. Three-Span Bent. Figure 7 represents a story of a three-span bent.



$M_A = -A.Wh$

$M_B = -B.Wh$

$M_C = -C.Wh$

$M_D = -D.Wh$

Fig. 7.

The general equations for this bent are:

$$\frac{\theta_A}{R}(6K_A + 2K) + \frac{\theta_B}{R} \cdot K = 6K_A \quad \ldots\ldots\ldots\ldots (1)$$

$$\frac{\theta_A}{R} \cdot K + \frac{\theta_B}{R}(6K_B + 4K) + \frac{\theta_C}{R} \cdot K = 6K_B \quad \ldots\ldots\ldots\ldots (2)$$

$$\frac{\theta_B}{R} \cdot K + \frac{\theta_C}{R}(6K_C + 4K) + \frac{\theta_D}{R} \cdot K = 6K_C \quad \ldots\ldots\ldots\ldots (3)$$

$$\frac{\theta_C}{R} \cdot K + \frac{\theta_D}{R}(6K_D + 2K) = 6K_D \quad \ldots\ldots\ldots\ldots (4)$$

TABLE III.

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Two-Span Bent.

(K of one member varying)

K_A	K_B	K_C	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{Wh}{6ER}$
1	1	1	0	1.000	1.000	1.000	0
1	1	1	1/2	.811	.649	.811	1.458
1	1	1	1	.6935	.461	.6935	2.304
1	1	1	3/2	.609	.348	.609	2.868
1	1	1	2	.545	.273	.545	3.274
1	1	1	∞	0	0	0	6.000
1	1	0	1	.680	.560	-.280	1.520
1	1	1/2	1	.690	.481	.504	2.154
1	1	1	1	.6935	.461	.6935	2.304
1	1	3/2	1	.694	.453	.776	2.378
1	1	2	1	.695	.448	.826	2.410
1	1	∞	1	.696	.430	1.000	2.558
1	0	1	1	.800	-.400	.800	.800
1	1/2	1	1	.721	.222	.721	1.894
1	1	1	1	.6935	.461	.6935	2.304
1	3/2	1	1	.676	.588	.676	2.532
1	2	1	1	.668	.668	.668	2.656
1	∞	1	1	.625	1.000	.625	3.250

TABLE IV.

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Two-Span Bent.

(K's of two columns varying simultaneously)

K_A	K_B	K_C	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{Wh}{6ER}$
0	1	0	1	-.333	.667	-.333	.667
1/2	1	1/2	1	.500	.500	.500	2.000
1	1	1	1	.6935	.461	.6935	2.304
3/2	1	3/2	1	.779	.445	.779	2.436
2	1	2	1	.825	.435	.825	2.530
∞	1	∞	1	1.000	.400	1.000	2.800
1	0	0	1	.778	-.222	.111	.444
1	1/2	1/2	1	.720	.247	.550	1.763
1	1	1	1	.6935	.461	.6935	2.304
1	3/2	3/2	1	.677	.581	.766	2.605
1	2	2	1	.668	.658	.810	2.792
1	∞	∞	1	.625	1.000	1.000	3.625

TABLE V.

Coefficients of Wh for the Two-Span Bent.

(K of one member varying)

K_A	K_B	K_C	K	A	B	C
1	1	1	0	.125	.250	.125
1	1	1	1/2	.1295	.241	.1295
1	1	1	1	.133	.234	.133
1	1	1	3/2	.136	.228	.136
1	1	1	2	.139	.222	.139
1	1	1	∞	.167	.167	.167
1	1	0	1	.210	.290	0
1	1	1/2	1	.144	.241	.115
1	1	1	1	.133	.234	.133
1	1	3/2	1	.129	.230	.141
1	1	2	1	.127	.229	.144
1	1	∞	1	.119	.223	.158
1	0	1	1	.250	0	.250
1	1/2	1	1	.147	.206	.147
1	1	1	1	.133	.234	.133
1	3/2	1	1	.128	.244	.128
1	2	1	1	.125	.250	.125
1	∞	1	1	.115	.270	.115

TABLE VI.

Coefficients of Wh for the Two-Span Bent.

(K's of two Columns varying simultaneously)

K_A	K_B	K_C	K	A	B	C
0	1	0	1	0	.500	0
1/2	1	1/2	1	.125	.250	.125
1	1	1	1	.133	.234	.133
3/2	1	3/2	1	.136	.228	.136
2	1	2	1	.138	.224	.138
∞	1	∞	1	.143	.214	.143
1	0	0	1	.500	0	0
1	1/2	1/2	1	.159	.213	.128
1	1	1	1	.133	.234	.133
1	3/2	3/2	1	.124	.241	.135
1	2	2	1	.119	.245	.136
1	∞	∞	1	.103	.259	.138

$$\frac{6ER}{Wh} = \frac{1}{2K_A(1 - \frac{\theta_B}{R}) + 2K_B(1 - \frac{\theta_B}{R}) + 2K_C(1 - \frac{\theta_C}{R}) + 2K_D(1 - \frac{\theta_D}{R})} \quad . \; . \; . \; (b)$$

The coefficients of Wh are:

$$A = \frac{6ER}{Wh} \cdot K_A(1 - \frac{\theta_A}{R})$$

$$B = \frac{6ER}{Wh} \cdot K_B(1 - \frac{\theta_B}{R})$$

$$C = \frac{6ER}{Wh} \cdot K_C(1 - \frac{\theta_C}{R})$$

$$D = \frac{6ER}{Wh} \cdot K_D(1 - \frac{\theta_D}{R})$$

The cases investigated are:

(I) $K_A = K_B = K_C = K_D = 1$, $K = 0, 1/10, 1/2, 1, 3/2, 2$ and ∞.

(II) $K_A = K_B = K_C = K = 1$, $K_D = 0, 1/2, 1, 3/2, 2$ and ∞.

(III) $K_A = K_B = K_D = K = 1$, $K_C = 0, 1/2$, etc.

(IV) $K_A = K_B = K = 1$, $K_C = K_D = 0, 1/2, 1, 3/2, 2$, and ∞.

(V) $K_A = K_C = K = 1$, $K_B = K_D = 0, 1/2$, etc.

(VI) $K_B = K_C = K = 1$, $K_A = K_D = 0, 1/2$, etc.

(VII) $K_A = K_D = K = 1$, $K_B = K_C = 0, 1/2$, etc.

The values of the $\frac{\theta}{R}$'s and $\frac{Wh}{6ER}$ are given in Tables VII and VIII, and the moments in the columns in terms of Wh are given in Tables IX and X. The changes in moments in the columns due to equal and simultaneous changes in the K's of the girders are shown graphically in Diagram 17, and changes in the moments in the columns due to changes in the K's of the columns are given in Diagrams 18 to 33 inclusive.

TABLE VII.

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Three-Span Bent.

(K's of two columns varying simultaneously.)

K_A	K_B	K_C	K_D	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{Wh}{6ER}$
1	1	1	1	0	1.000	1.000	1.000	1.000	0
1	1	1	1	1/10	.953	.907	.907	.953	.560
1	1	1	1	1/2	.811	.659	.659	.811	2.120
1	1	1	1	1	.690	.483	.483	.690	3.308
1	1	1	1	3/2	.605	.378	.378	.605	4.068
1	1	1	1	2	.538	.308	.308	.538	4.616
1	1	1	1	∞	0	0	0	0	8.000
1	1	1	0	1	.690	.473	.581	-.291	2.512
1	1	1	1/2	1	.690	.481	.502	.500	3.154
1	1	1	1	1	.690	.483	.483	.690	3.308
1	1	1	3/2	1	.690	.484	.474	.775	3.379
1	1	1	2	1	.690	.484	.470	.825	3.412
1	1	1	∞	1	.689	.485	.451	1.000	3.567
1	1	0	1	1	.679	.565	-.339	.791	1.930
1	1	1/2	1	1	.687	.506	.254	.719	2.922
1	1	1	1	1	.690	.483	.483	.690	3.308
1	1	3/2	1	1	.691	.470	.605	.674	3.515
1	1	2	1	1	.691	.462	.679	.665	3.648
1	1	∞	1	1	.696	.430	1.000	.625	4.183

TABLE VIII

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Three-Span Bent.

(K's of two columns varying simultaneously.)

K_A	K_B	K_C	K_D	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{Wh}{6ER}$
1	1	0	0	1	.681	.546	-.156	.078	1.546
1	1	1/2	1/2	1	.686	.504	.278	.544	2.798
1	1	1	1	1	.690	.483	.483	.690	3.308
1	1	3/2	3/2	1	.691	.470	.596	.763	3.601
1	1	2	2	1	.692	.464	.670	.808	3.796
1	1	∞	∞	1	.696	.430	1.000	1.000	4.558
1	0	1	0	1	.796	-.366	.670	-.335	1.068
1	1/2	1	1/2	1	.719	.250	.526	.495	2.765
1	1	1	1	1	.690	.483	.483	.690	3.308
1	3/2	1	3/2	1	.674	605	.461	.775	3.590
1	2	1	2	1	.664	.680	.449	.825	3.754
1	∞	1	∞	1	.625	1.000	.400	1.000	4.625
0	1	1	0	1	-.286	.571	.571	-.286	1.716
1/2	1	1	1/2	1	.500	.500	.500	.500	3.000
1	1	1	1	1	.690	.483	.483	.690	3.308
3/2	1	1	3/2	1	.776	.475	.475	.776	3.444
2	1	1	2	1	.823	.470	.470	.823	3.536
∞	1	1	∞	1	1.000	.455	.455	1.000	3.817
1	0	0	1	1	769	-.154	- .154	.769	.924
1	1/2	1/2	1	1	.714	.286	.286	.714	2.572
1	1	1	1	1	.690	.483	.483	.690	3.308
1	3/2	3/2	1	1	.676	.595	.595	.676	3.726
1	2	2	1	1	.667	.667	.667	.667	4.000
1	∞	∞	1	1	.625	1.000	1.000	.625	5.250

TABLE IX.

Coefficients of Wh for the Three-Span Bent.

(K of one member varying)

K_A	K_B	K_C	K_D	K	A	B	C	D
1	1	1	1	0	.083	.167	.167	.083
1	1	1	1	1/10	.084	.166	.166	.084
1	1	1	1	1/2	.089	.161	.161	.089
1	1	1	1	1	.094	.156	.156	.094
1	1	1	1	3/2	.097	.153	.153	.097
1	1	1	1	2	.100	.150	.150	.100
1	1	1	1	∞	.125	.125	.125	.125
1	1	1	0	1	.123	.210	.167	0
1	1	1	1/2	1	.098	.165	.158	.079
1	1	1	1	1	.094	.156	.156	.094
1	1	1	3/2	1	092	.153	.155	.100
1	1	1	2	1	.091	.151	.155	.103
1	1	1	∞	1	.087	.144	.154	.115
1	1	0	1	1	.166	.226	0	.108
1	1	1/2	1	1	.107	.169	.128	.096
1	1	1	1	1	.094	156	.156	.094
1	1	3/2	1	1	.088	.151	.168	.093
1	1	2	1	1	085	.147	.176	.092
1	1	∞	1	1	.073	.135	.201	.090

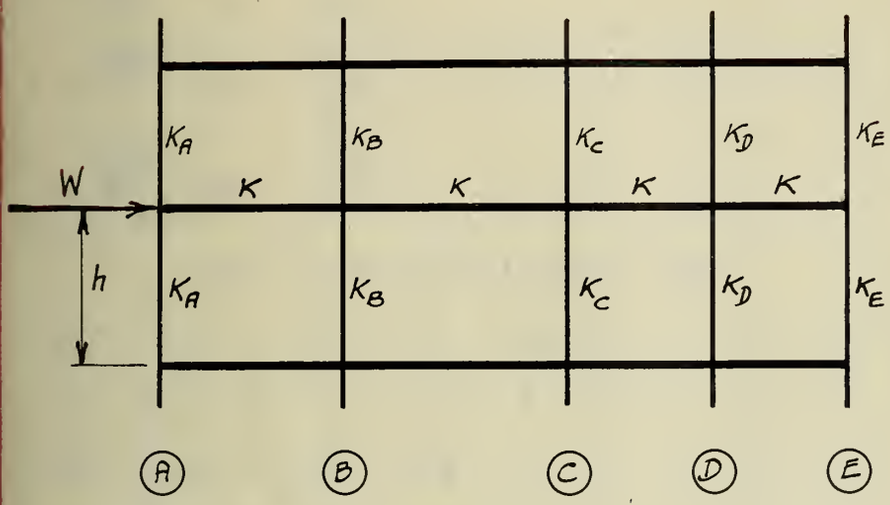
TABLE X.

Coefficients of Wh for the Three-Span Bent.

(K's of two columns varying simultaneously)

K_A	K_B	K_C	K_D	K	A	B	C	D
1	1	0	0	1	.206	.294	0	0
1	1	1/2	1/2	1	.112	.178	.129	.081
1	1	1	1	1	.094	.156	.156	.094
1	1	3/2	3/2	1	.086	.147	.168	.099
1	1	2	2	1	.084	.141	.174	.101
1	1	∞	∞	1	.066	.125	.199	.110
1	0	1	0	1	.191	0	.309	0
1	1/2	1	1/2	1	.102	.136	.171	.091
1	1	1	1	1	.094	.156	.156	.094
1	3/2	1	3/2	1	.091	.165	.150	.094
1	2	1	2	1	.090	.170	.147	.095
1	∞	1	∞	1	.081	.181	.130	.108
0	1	1	0	1	0	.250	.250	0
1/2	1	1	1/2	1	.083	.167	.167	.083
1	1	1	1	1	.094	.156	.156	.094
3/2	1	1	3/2	1	.098	.152	.152	.098
2	1	1	2	1	.100	.150	.150	.100
∞	1	1	∞	1	.107	.143	.143	.107
1	0	0	1	1	.250	0	0	.250
1	1/2	1/2	1	1	.111	.139	.139	.111
1	1	1	1	1	.094	.156	.156	.094
1	3/2	3/2	1	1	.087	.163	.163	.087
1	2	2	1	1	.083	.167	.167	.083
1	∞	∞	1	1	.072	.178	.178	.072

12. Four-Span Bent. Figure 8 represents a story of a four-span bent.



$M_A = -A.Wh$

$M_B = -B.Wh$

$M_C = -C.Wh$

$M_D = -D.Wh$

$M_E = -E.Wh$

Fig. 8.

The general equations for this bent are:

$$\frac{\theta_A}{R}(6K_A + 2K) + \frac{\theta_B}{R} \cdot K = 6K_A \quad \ldots\ldots\ldots\ldots (1)$$

$$\frac{\theta_A}{R} \cdot K + \frac{\theta_B}{R}(6K_B + 4K) + \frac{\theta_C}{R} \cdot K = 6K_B \quad \ldots\ldots\ldots\ldots (2)$$

$$\frac{\theta_B}{R} \cdot K + \frac{\theta_C}{R}(6K_C + 4K) + \frac{\theta_D}{R} \cdot K = 6K_C \quad \ldots\ldots\ldots\ldots (3)$$

$$\frac{\theta_C}{R} \cdot K + \frac{\theta_D}{R}(6K_D + 4K) + \frac{\theta_E}{R} \cdot K = 6K_D \quad \ldots\ldots\ldots\ldots (4)$$

$$\frac{\theta_D}{R} \cdot K + \frac{\theta_E}{R}(6K_E + 2K) = 6K_E \quad \ldots\ldots\ldots\ldots (5)$$

$$\frac{6ER}{Wh} = \frac{1}{2K_A(1 - \frac{\theta_A}{R}) + 2K_B(1 - \frac{\theta_B}{R}) + 2K_C(1 - \frac{\theta_C}{R}) + 2K_D(1 - \frac{\theta_D}{R}) + 2K_E(1 - \frac{\theta_E}{R})} \quad .(6)$$

The coefficients of Wh are:

$$A = \frac{6ER}{Wh} \cdot K_A(1 - \frac{\theta_A}{R})$$

$$B = \frac{6ER}{Wh} \cdot K_B\left(1 - \frac{\theta_B}{R}\right)$$

$$C = \frac{6ER}{Wh} \cdot K_C\left(1 - \frac{\theta_C}{R}\right)$$

$$D = \frac{6ER}{Wh} \cdot K_D\left(1 - \frac{\theta_D}{R}\right)$$

$$E = \frac{6ER}{Wh} \cdot K_E\left(1 - \frac{\theta_E}{R}\right)$$

The cases investigated are:

(I) $K_A = K_B = K_C = K_D = K_E = 1$, $K = 0, 1/2, 1, 2$, and ∞.

(II) $K_A = K_B = K_C = K_D = K = 1$ $K_E = 0, 1/2, 1, 2$, and ∞.

(III) $K_A = K_B = K_C = K_E = K = 1$ $K_D = 0, 1/2$, etc.

(IV) $K_A = K_B = K_D = K_E = K = 1$, $K_C = 0, 1/2$, etc.

(V) $K_A = K_B = K_C = K = 1$, $K_D = K_E = 0, 1/2, 1, 2$, and ∞.

(VI) $K_A = K_B = K_D = K = 1$, $K_C = K_E = 0, 1/2$, etc.

(VII) $K_A = K_C = K_D = K = 1$, $K_B = K_E = 0, 1/2$, etc.

(VIII) $K_B = K_C = K_D = K = 1$, $K_A = K_E = 0, 1/2$, etc.

(IX) $K_A = K_B = K_E = K = 1$, $K_C = K_D = 0, 1/2$, etc.

(X) $K_A = K_C = K_E = K = 1$, $K_B = K_D = 0, 1/2$, etc.

The values of the $\frac{\theta}{R}$'s and $\frac{Wh}{6ER}$ are given in Tables XI and XII, and the moments in the columns in terms of Wh are given in Tables XIII and XIV. The changes in moments in the columns due to equal and simultaneous changes in the K's of the girders are shown graphically in Diagram 34, and changes in the moments in the columns due to changes in the K's of the columns are given in Diagrams 35 to 61 inclusive.

TABLE XI

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Four-Span Bent.

(K of one member varying.)

K_A	K_B	K_C	K_D	K_E	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{\theta_E}{R}$	$\frac{Wh}{6ER}$
1	1	1	1	1	0	1.000	1.000	1.000	1.000	1.000	0
1	1	1	1	1	1/2	.810	.657	.667	.657	.810	2.798
1	1	1	1	1	1	.690	.480	.504	.480	.690	4.312
1	1	1	1	1	2	.540	.303	.342	.303	.540	5.944
1	1	1	1	1	∞	0	0	0	0	0	10.000
1	1	1	1	0	1	.690	.481	.494	.580	-.290	3.510
1	1	1	1	1/2	1	.690	.481	.501	.500	.500	4.156
1	1	1	1	1	1	.690	.480	.504	.480	.690	4.312
1	1	1	1	2	1	.691	.481	.506	.467	.825	4.410
1	1	1	1	∞	1	.690	.480	.507	.449	1.000	4.564
1	1	1	0	1	1	.691	.472	.586	-.345	.794	2.914
1	1	1	1/2	1	1	.691	.479	.526	.251	.720	3.917
1	1	1	1	1	1	.690	.480	.504	.480	.690	4.312
1	1	1	2	1	1	.690	.482	.484	.678	.665	4.646
1	1	1	∞	1	1	.689	.486	.451	1.000	.625	5.190
1	1	0	1	1	1	.681	.560	-.280	.560	.681	3.036
1	1	1/2	1	1	1	.686	.502	.284	.502	.686	3.964
1	1	1	1	1	1	.690	.480	.504	.480	.690	4.312
1	1	2	1	1	1	.692	.461	.692	.461	.692	4.620
1	1	∞	1	1	1	.696	.430	1.000	.430	.696	5.116

TABLE XII.

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Four-Span Bent.

(K's of two columns varying simultaneously)

K_A	K_B	K_C	K_D	K_E	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{\theta_E}{R}$	$\frac{Wh}{6ER}$
1	1	1	0	0	1	.691	.474	.569	-.163	.081	2.532
1	1	1	1/2	1/2	1	.690	.478	.525	276	.544	3.794
1	1	1	1	1	1	.690	.480	.504	.480	.690	4.312
1	1	1	2	2	1	.689	.482	.484	.668	.809	4.782
1	1	1	∞	∞	1	.689	.486	.451	1.000	1.000	5.565
1	1	0	1	0	1	.680	.531	-.306	.664	-.332	2.250
1	1	1/2	1	1/2	1	.686	.504	.282	.523	.496	3.796
1	1	1	1	1	1	.690	.480	.504	.480	.690	4.312
1	1	2	1	2	1	.692	.461	.694	.448	.826	4.658
1	1	∞	1	∞	1	.696	.430	1.000	.400	1.000	5.358
1	0	1	1	0	1	.793	-.342	.576	.571	-.285	2.120
1	1/2	1	1	1/2	1	.718	.250	.525	.497	.500	3.770
1	1	1	1	1	1	.690	.480	.504	.480	.690	4.312
1	2	1	1	2	1	.666	.678	.486	.469	.825	4.746
1	∞	1	1	∞	1	.625	1.000	.455	.455	1.000	5.441
0	1	1	1	0	1	-.290	.581	.484	.581	-.290	2.708
1/2	1	1	1	1/2	1	.500	.500	.500	.500	.500	4.000
1	1	1	1	1	1	.690	.480	.504	.480	.690	4.312
2	1	1	1	2	1	.824	.467	.506	.467	.824	4.528
∞	1	1	1	∞	1	1.000	.449	.510	.449	1.000	4.817
1	1	0	0	1	1	.681	.541	-.093	-.170	.771	2.014
1	1	1/2	1/2	1	1	.688	.500	.318	.282	.715	3.594
1	1	1	1	1	1	.690	.480	.504	.480	.690	4.312
1	1	2	2	1	1	.692	.462	.680	.666	.667	4.974
1	1	∞	∞	1	1	.696	.430	1.000	1.000	.625	6.183
1	0	1	0	1	1	.796	-.367	.673	-.367	.796	1.470
1	1/2	1	1/2	1	1	.720	.248	.551	.248	.720	3.522
1	1	1	1	1	1	.690	.480	.504	.480	.690	4.312
1	2	1	2	1	1	.665	.679	.464	.679	.665	4.980
1	∞	1	∞	1	1	.625	1.000	.400	1.000	.625	6.050

TABLE XIII

Coefficients of Wh for the Four-Span Bent.

(K of one member varying)

K_A	K_B	K_C	K_D	K_E	K	A	B	C	D	E
1	1	1	1	1	0	.0625	.1250	.1250	.1250	.0625
1	1	1	1	1	1/2	.0679	.1227	.1190	.1227	.0679
1	1	1	1	1	1	.0719	.1205	.1149	.1205	.0719
1	1	1	1	1	2	.0774	.1173	.1109	.1173	.0774
1	1	1	1	1	∞	.1000	.1000	.1000	.1000	.1000
1	1	1	1	0	1	.0884	.1479	.1441	.1197	.0
1	1	1	1	1/2	1	.0745	.1250	.1200	.1202	.0600
1	1	1	1	1	1	.0719	.1205	.1149	.1205	.0719
1	1	1	1	2	1	.0700	.1178	.1120	.1208	0794
1	1	1	1	∞	1	.0680	.1139	.1080	.1209	.0895
1	1	1	0	1	1	.1060	.1812	.1421	0	.0707
1	1	1	1/2	1	1	.0790	.1331	.1210	.0956	.0715
1	1	1	1	1	1	.0719	.1205	.1149	.1205	.0719
1	1	1	2	1	1	.0667	.1116	.1110	.1387	.0721
1	1	1	∞	1	1	.0599	.0991	.1058	.1630	.0722
1	1	0	1	1	1	.1050	.1450	0	.1450	.1050
1	1	1/2	1	1	1	.0791	.1256	.0903	.1256	.0791
1	1	1	1	1	1	.0719	.1205	.1149	.1205	.0719
1	1	2	1	1	1	.0666	.1167	.1332	.1167	.0666
1	1	∞	1	1	1	.0594	.1114	.1563	.1114	.0594

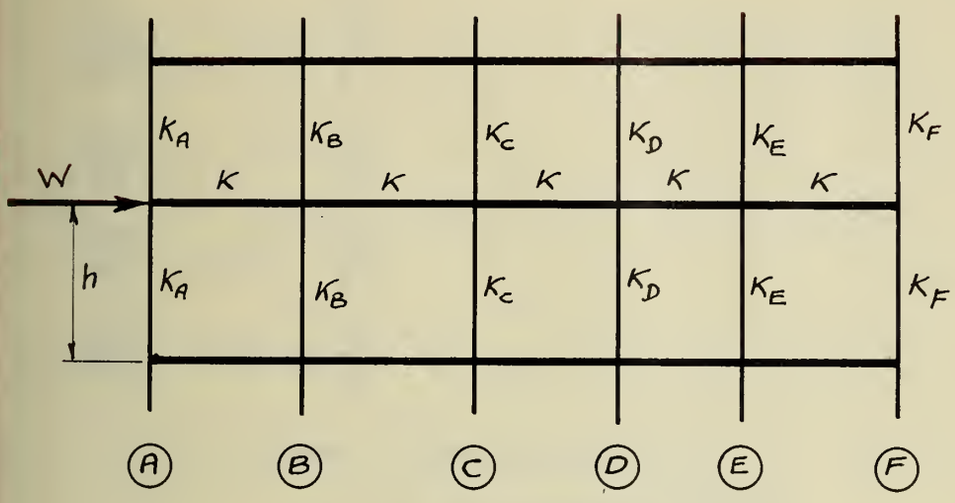
TABLE XIV

Coefficients of Wh for the Four-Span Bent.

(K's of two columns varying simultaneously)

K_A	K_B	K_C	K_D	K_E	K	A	B	C	D	E
1	1	1	0	0	1	.1221	.2080	.1701	0	0
1	1	1	1/2	1/2	1	.0818	.1378	.1251	.0955	.0601
1	1	1	1	1	1	.0719	.1205	.1149	.1205	.0719
1	1	1	2	2	1	.0651	.1083	.1080	.1390	.0799
1	1	1	∞	∞	1	.0558	.0924	.0987	.1630	.0899
1	1	0	1	0	1	.1421	.2087	0	.1493	0
1	1	1/2	1	1/2	1	.0827	.1307	.0946	.1257	.0664
1	1	1	1	1	1	.0719	.1205	.1149	.1205	.0719
1	1	2	1	2	1	.0652	.1141	.1298	.1170	.0737
1	1	∞	1	∞	1	.0568	.1064	.1502	.1120	.0746
1	0	1	1	0	1	.0976	0	.2000	.2025	0
1	1/2	1	1	1/2	1	.0748	.0995	.1260	.1335	.0664
1	1	1	1	1	1	.0719	.1205	.1149	.1205	.0719
1	2	1	1	2	1	.0704	.1357	.1083	.1119	.0738
1	∞	1	1	∞	1	.0689	.1556	.1002	.1002	.0752
0	1	1	1	0	1	.0	.1549	.1903	.1549	0
1/2	1	1	1	1/2	1	.0625	.1250	.1250	.1250	.0625
1	1	1	1	1	1	.0719	.1205	.1149	.1205	.0719
2	1	1	1	2	1	.0778	.1178	.1091	.1178	.0778
∞	1	1	1	∞	1	.0847	.1144	.1017	.1144	.0847
1	1	0	0	1	1	.1584	.2279	0	0	.1137
1	1	1/2	1/2	1	1	.0869	.1391	.0949	.0999	.0793
1	1	1	1	1	1	.0719	.1205	.1149	.1205	.0719
1	1	2	2	1	1	.0619	.1081	.1288	.1342	.0670
1	1	∞	∞	1	1	.0492	.0922	.1464	.1518	.0606
1	0	1	0	1	1	.1388	0	.2224	0	.1388
1	1/2	1	1/2	1	1	.0795	.1068	.1275	.1068	.0795
1	1	1	1	1	1	.0719	.1205	.1149	.1205	.0719
1	2	1	2	1	1	.0673	.1289	.1077	.1289	.0673
1	∞	1	∞	1	1	.0620	.1384	.0991	.1384	.0620

13. Five-Span Bent. Figure 9 represents a story of a five-span bent.



$M_A = - A.Wh$

$M_B = - B.Wh$

$M_C = - C.Wh$

$M_D = - D.Wh$

$M_E = - E.Wh$

$M_F = - F.Wh$

Fig. 9.

The general equations for this bent are:

$$\frac{\theta_A}{R}(6K_A + 2K) + \frac{\theta_B}{R} \cdot K = 6K_A \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$\frac{\theta_A}{R} \cdot K + \frac{\theta_B}{R}(6K_B + 4K) + \frac{\theta_C}{R} \cdot K = 6K_B \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{\theta_B}{R} \cdot K + \frac{\theta_C}{R}(6K_C + 4K) + \frac{\theta_D}{R} \cdot K = 6K_C \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$\frac{\theta_C}{R} \cdot K + \frac{\theta_D}{R}(6K_D + 4K) + \frac{\theta_E}{R} \cdot K = 6K_D \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$\frac{\theta_D}{R} \cdot K + \frac{\theta_E}{R}(6K_E + 4K) + \frac{\theta_F}{R} \cdot K = 6K_E \quad \ldots\ldots\ldots\ldots \quad (5)$$

$$\frac{\theta_E}{R} \cdot K + \frac{\theta_F}{R}(6K_F + 2K) = 6K_F \quad \ldots\ldots\ldots\ldots \quad (6)$$

$$\frac{6ER}{Wh} = \frac{1}{2K_A(1-\frac{\theta_A}{R})+2K_B(1-\frac{\theta_B}{R})+2K_C(1-\frac{\theta_C}{R})+2K_D(1-\frac{\theta_D}{R})+2K_E(1-\frac{\theta_E}{R})+2K_F(1-\frac{\theta_F}{R})}$$

The coefficients of Wh are:

$$A = \frac{6ER}{Wh} \cdot K_A(1 - \frac{\theta_A}{R})$$

$$B = \frac{6ER}{Wh} \cdot K_B(1 - \frac{\theta_B}{R})$$

$$C = \frac{6ER}{Wh} \cdot K_C(1 - \frac{\theta_C}{R})$$

$$D = \frac{6ER}{Wh} \cdot K_D(1 - \frac{\theta_D}{R})$$

$$E = \frac{6ER}{Wh} \cdot K_E(1 - \frac{\theta_E}{R})$$

$$F = \frac{6ER}{Wh} \cdot K_F(1 - \frac{\theta_F}{R})$$

The cases investigated are:

(a) K of one member varying.

(I) $K_A = K_B = K_C = K_D = K_E = K_F = 1$, $K = 0, 1/2, 1, 3/2, 2$, and ∞.

(II) $K_A = K_B = K_C = K_D = K_E = K = 1$, $K_F = 0, 1/2, 1, 3/2, 2$, and ∞.

(III) $K_A = K_B = K_C = K_D = K_F = K = 1$, $K_E = 0, 1/2$, etc.

(IV) $K_A = K_B = K_C = K_E = K_F = K = 1$, $K_D = 0, 1/2$, etc.

(b) K of 2 columns varying simultaneously.

(V) $K_A = K_B = K_C = K_D = K = 1$, $K_E = K_F = 0, 1/2, 1, 3/2, 2$, and ∞.

(VI) $K_A = K_B = K_C = K_E = K = 1$, $K_D = K_F = 0, 1/2$, etc.

(VII) $K_A = K_B = K_D = K_E = K = 1$, $K_C = K_F = 0, 1/2$, etc.

(VIII) $K_A = K_C = K_D = K_E = K = 1$, $K_B = K_F = 0, 1/2$, etc.

(IX) $K_B = K_C = K_D = K_E = K = 1$, $K_A = K_F = 0, 1/2$, etc.

(X) $K_A = K_B = K_C = K_F = K = 1$, $K_D = K_E = 0, 1/2$, etc.

(XI) $K_A = K_B = K_D = K_F = K = 1$, $K_C = K_E = 0, 1/2$, etc.

(XII) $K_A = K_C = K_D = K_F = K = 1$, $K_B = K_E = 0, 1/2$, etc.

(XIII) $K_A = K_B = K_E = K_F = K = 1$, $K_C = K_D = 0, 1/2$, etc.

(c) K of 3 columns varying simultaneously.

(XIV) $K_A = K_B = K_C = K = 1$, $K_D = K_E = K_F = 0, 1/2, 1, 3/2, 2$, and ∞.

(XV) $K_A = K_B = K_D = K = 1$, $K_C = K_E = K_F = 0$, 1/2, etc.

(XVI) $K_A = K_C = K_D = K = 1$, $K_B = K_E = K_F = 0$, 1/2, etc.

(XVII) $K_B = K_C = K_D = K = 1$, $K_A = K_E = K_F = 0$, 1/2, etc.

(XVIII) $K_A = K_B = K_E = K = 1$, $K_C = K_D = K_F = 0$, 1/2, etc.

(XIX) $K_A = K_C = K_E = K = 1$, $K_B = K_D = K_F = 0$, 1/2, etc.

(XX) $K_B = K_C = K_E = K = 1$, $K_A = K_D = K_F = 0$, 1/2, etc.

(XXI) $K_A = K_D = K_E = K = 1$, $K_B = K_C = K_F = 0$, 1/2, etc.

(XXII) $K_A = K_B = K_F = K = 1$, $K_C = K_D = K_E = 0$, 1/2, etc.

(XXIII) $K_A = K_D = K_F = K = 1$, $K_B = K_C = K_E = 0$, 1/2, etc.

The values of the $\frac{\theta}{R}$'s and $\frac{Wh}{6ER}$ are given in Tables XV, XVI, and XVII, and the moments in the columns in terms of Wh are given in Tables XVIII, XIX, and XX. The changes in moments in the columns due to equal and simultaneous changes in the K's of the girders are shown graphically in Diagram 62, and changes in the moments in the columns due to changes in the K's of the columns are given in Diagrams 63 to 150 inclusive.

TABLE XV

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Five-Span Bent.

(K of one member varying.)

K_A	K_B	K_C	K_D	K_E	K_F	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{\theta_E}{R}$	$\frac{\theta_F}{R}$	$\frac{Wh}{6ER}$
1	1	1	1	1	1	0	1.000	1.000	1.000	1.000	1.000	1.000	0
1	1	1	1	1	1	1/2	.811	.659	.669	.669	.659	.811	3.444
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	1	1	1	1	3/2	.605	.374	.404	.404	.374	.605	6.468
1	1	1	1	1	1	2	.539	.303	.335	.335	.303	.539	7.292
1	1	1	1	1	1	∞	0	0	0	0	0	0	12.000
1	1	1	1	1	0	1	.690	.481	.503	.491	.580	-.290	4.510
1	1	1	1	1	1/2	1	.691	.481	.502	.500	.500	.500	5.152
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	1	1	1	3/2	1	.690	.481	.502	.502	.472	.775	5.381
1	1	1	1	1	2	1	.690	.481	.501	.503	.468	.824	5.418
1	1	1	1	1	∞	1	.690	.481	.501	.505	.449	1.000	5.564
1	1	1	1	0	1	1	.691	.482	.494	.586	-.344	.795	3.904
1	1	1	1	1/2	1	1	.690	.481	.499	.525	.251	.719	4.921
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	690	5.308
1	1	1	1	3/2	1	1	.690	.480	.503	.489	.603	.675	5.517
1	1	1	1	2	1	1	.690	.480	.504	.481	.678	.665	5.648
1	1	1	1	∞	1	1	.690	.480	.507	.449	1.000	.625	6.189
1	1	1	0	1	1	1	.691	.472	.581	-.285	.560	.679	4.034
1	1	1	1/2	1	1	1	.690	.479	.524	.282	503	.687	4.952
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	1	3/2	1	1	1	.689	.481	.489	.618	.469	.691	5.508
1	1	1	2	1	1	1	.689	.482	.482	.690	.461	.692	5.628
1	1	1	∞	1	1	1	.685	.482	.448	1.000	.430	.696	6.144

TABLE XVI.

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Five-Span Bent.

(K's of two columns varying simultaneously.)

K_A	K_B	K_C	K_D	K_E	K_F	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{\theta_E}{R}$	$\frac{\theta_F}{R}$	$\frac{Wh}{6ER}$
1	1	1	1	0	0	1	.690	.481	.495	.566	-.162	.081	3.536
1	1	1	1	1/2	1/2	1	.690	.481	.501	.523	.276	.546	4.788
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	1	1	3/2	3/2	1	.690	.480	.502	.490	.595	.764	5.599
1	1	1	1	2	2	1	.690	.480	.504	.483	.669	.809	5.774
1	1	1	1	∞	∞	1	.691	.481	.508	.450	1.000	1.000	6.557
1	1	1	0	1	0	1	.691	.472	.584	-.312	.665	-.332	3.176
1	1	1	1/2	1	1/2	1	.690	.478	.524	.279	.523	.495	4.796
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	1	3/2	1	3/2	1	.690	.483	.489	.620	.461	.776	5.566
1	1	1	2	1	2	1	.690	.483	.483	.691	.449	.825	5.726
1	1	1	∞	1	∞	1	.689	.486	.451	1.000	.400	1.000	6.365
1	1	0	1	1	0	1	.680	.561	-.283	571	.571	-.286	3.234
1	1	1/2	1	1	1/2	1	.687	.503	.282	.522	.497	.500	4.800
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	3/2	1	1	3/2	1	.691	.469	.619	.490	.473	.775	5.572
1	1	2	1	1	2	1	.693	.462	.691	.484	.470	.824	5.722
1	1	∞	1	1	∞	1	.696	.430	1.000	.455	.455	1.000	6.374
1	0	1	1	1	0	1	.793	-.344	.585	.483	.581	-.290	3.116
1	1/2	1	1	1	1/2	1	.717	.250	.524	.497	.500	.500	4.774
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	3/2	1	1	1	3/2	1	.673	.602	.488	.503	.471	.774	5.602
1	2	1	1	1	2	1	.666	.679	.482	.506	.467	.825	5.742
1	∞	1	1	1	∞	1	.625	1.000	.449	.510	.449	1.000	6.441
0	1	1	1	1	0	1	-.290	.581	.493	.493	.581	-.290	3.704
1/2	1	1	1	1	1/2	1	.500	.500	.500	.500	.500	.500	5.000
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
3/2	1	1	1	1	3/2	1	.774	.471	.501	.501	.471	774	5.468
2	1	1	1	1	2	1	.824	.467	.503	.503	.467	.824	5.528
∞	1	1	1	1	∞	1	1.000	.450	.505	.505	.450	1.000	5.814
1	1	1	0	0	1	1	.690	.475	.562	- 099	-.168	.771	3.004
1	1	1	1/2	1/2	1	1	.690	.479	.521	.314	.281	.714	4.597
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	1	3/2	3/2	1	1	.689	.481	.490	.608	.593	.675	5.727
1	1	1	2	2	1	1	.689	.482	.484	.677	.665	.666	5.990
1	1	1	∞	∞	1	1	.689	.485	451	1.000	1.000	.625	7.192
1	1	0	1	0	1	1	.679	.562	-.307	.666	-.366	.796	2.594
1	1	1/2	1	1/2	1	1	.687	.504	.278	.548	.248	.719	4.558
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	3/2	1	3/2	1	1	.691	.469	.620	.477	.604	.675	5.704
1	1	2	1	2	1	1	.692	.461	.693	.462	.680	.665	5.948
1	1	∞	1	∞	1	1	.696	.430	1.000	.400	1.000	.625	6.983
1	0	1	1	0	1	1	.792	-.342	.576	.576	-.342	.792	2.528
1	1/2	1	1	1/2	1	1	.719	.251	.522	.522	.251	.719	4.534
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	3/2	1	1	3/2	1	1	.674	.602	.491	.491	.602	.674	5.728
1	2	1	1	2	1	1	.665	.679	.484	.484	.679	.665	5.972
1	∞	1	1	∞	1	1	.625	1.000	.455	.455	1.000	.625	7.066

(Continued)

TABLE XVI (Cont'd.)

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Five-Span Bent.

(K's of two columns varying simultaneously.)

K_A	K_B	K_C	K_D	K_E	K_F	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{\theta_E}{R}$	$\frac{\theta_F}{R}$	$\frac{Wh}{6ER}$
1	1	0	0	1	1	1	.682	.542	-.1085	-.108	.542	.682	3.104
1	1	1/2	1/2	1	1	1	.686	.500	.312	.312	.500	.686	4.636
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	3/2	3/2	1	1	1	.690	.469	.609	.609	.469	.690	5.710
1	1	2	2	1	1	1	.691	.462	.677	.677	.462	.691	5.972
1	1	∞	∞	1	1	1	.696	.430	1.000	1.000	.430	.696	7.116

TABLE XVII

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Five-Span Bent.

(K's of three columns varying simultaneously.)

K_A	K_B	K_C	K_D	K_E	K_F	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{\theta_E}{R}$	$\frac{\theta_F}{R}$	$\frac{Wh}{6ER}$
1	1	1	0	0	0	1	.690	.474	.567	-.153	.437	-.218	2.538
1	1	1	1/2	1/2	1/2	1	.690	.478	.521	.310	.308	.539	4.465
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	1	3/2	3/2	3/2	1	.690	.481	.490	.609	.586	.764	5.801
1	1	1	2	2	2	1	.690	.482	.483	.679	.656	.810	6.112
1	1	1	∞	∞	∞	1	.690	.486	.451	1.000	1.000	1.000	7 563
1	1	0	1	0	0	1	.680	.562	-.303	.648	-.185	.093	2.220
1	1	1/2	1	1/2	1/2	1	.686	.504	.279	.545	.273	.545	4.433
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	3/2	1	3/2	3/2	1	.691	.469	.619	.478	.596	.764	5.787
1	1	2	1	2	2	1	.692	.461	.692	.464	.670	.809	6.082
1	1	∞	1	∞	∞	1	.696	.430	1.000	.400	1.000	1.000	7.358
1	0	1	1	0	0	1	.792	-.343	.578	.555	-.159	.080	2.150
1	1/2	1	1	1/2	1/2	1	.720	.251	.523	.521	.277	.545	4.399
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	3/2	1	1	3/2	3/2	1	.675	.603	.491	.491	.596	.765	5.794
1	2	1	1	2	2	1	.666	.678	.483	.485	.669	.809	6.108
1	∞	1	1	∞	∞	1	.625	1.000	.455	.455	1.000	1.000	7.442
0	1	1	1	0	0	1	-.290	.580	.485	.568	-.162	.081	2.734
1/2	1	1	1	1/2	1/2	1	.500	.500	.498	.523	.276	.545	4.637
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
3/2	1	1	1	3/2	3/2	1	.775	.472	.504	.490	.596	.764	5.663
2	1	1	1	2	2	1	.826	.468	.506	.483	.670	.811	5.858
∞	1	1	1	∞	∞	1	1.000	.449	.510	.449	1.000	1.000	6.817

(Continued)

TABLE XVII (Cont'd).

Values of $\frac{\theta}{R}$ and $\frac{Wh}{6ER}$ for the Five-Span Bent.

(K's of three columns varying simultaneously.)

K_A	K_B	K_C	K_D	K_E	K_F	K	$\frac{\theta_A}{R}$	$\frac{\theta_B}{R}$	$\frac{\theta_C}{R}$	$\frac{\theta_D}{R}$	$\frac{\theta_E}{R}$	$\frac{\theta_F}{R}$	$\frac{Wh}{6ER}$
1	1	0	0	1	0	1	.682	.542	-.101	-.136	.646	-.523	2.280
1	1	1/2	1/2	1	1/2	1	.688	.500	.313	.309	.520	.496	4.406
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	3/2	3/2	1	3/2	1	.691	.470	.609	.610	.461	.776	5.771
1	1	2	2	1	2	1	.692	.462	.678	.679	.449	.825	6.066
1	1	∞	∞	1	∞	1	.696	.430	1.000	1.000	.400	1.000	7.358
1	0	1	0	1	0	1	.796	-.366	.670	-.334	.667	-.333	1.734
1	1/2	1	1/2	1	1/2	1	.719	.248	.548	.276	.522	.495	4.403
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	3/2	1	3/2	1	3/2	1	.675	.604	.478	.620	.461	.776	5.772
1	2	1	2	1	2	1	.666	.680	.463	.694	.449	.826	6.044
1	∞	1	∞	1	∞	1	.625	1.000	.400	1.000	.400	1.000	7.225
0	1	1	0	1	0	1	-.285	.571	.274	-.309	.654	-.372	2.382
1/2	1	1	1/2	1	1/2	1	.561	.498	.522	.279	.523	.498	4.638
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
3/2	1	1	3/2	1	3/2	1	.775	.473	.491	.620	.460	.776	5.639
2	1	1	2	1	2	1	.824	.469	.484	.691	.448	.824	5.842
∞	1	1	∞	1	∞	1	1.000	.455	.455	1.000	.400	1.000	6.617
1	0	0	1	1	0	1	.770	-.169	-.096	.552	.573	-.287	2.210
1	1/2	1/2	1	1	1/2	1	.715	.282	.314	.518	.498	.500	4.442
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	3/2	3/2	1	1	3/2	1	.676	.594	.609	.491	.473	.773	5.792
1	2	2	1	1	2	1	.666	.666	.667	.485	.469	.823	6.096
1	∞	∞	1	1	∞	1	.625	1.000	1.000	.455	.455	1.000	7.442
1	1	0	0	0	1	1	.681	.548	-.161	.694	-.216	.777	1.988
1	1	1/2	1/2	1/2	1	1	.685	.500	.308	.545	.277	.715	4.264
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	1	3/2	3/2	3/2	1	1	.692	.470	.611	.600	.595	.676	5.906
1	1	2	2	2	1	1	.693	.463	.680	.686	.667	.667	6.302
1	1	∞	∞	∞	1	1	.696	.430	1.000	1.000	1.000	.625	8.183
1	0	0	1	0	1	1	.770	-.162	-.122	.646	-.361	.795	1.574
1	1/2	1/2	1	1/2	1	1	.716	.283	.311	.545	.248	.720	4.195
1	1	1	1	1	1	1	.690	.481	.502	.502	.481	.690	5.308
1	3/2	3/2	1	3/2	1	1	.676	.594	.610	.478	.604	.675	5.918
1	2	2	1	2	1	1	.667	.666	.680	.464	.680	.665	6.304
1	∞	∞	1	∞	1	1	.625	1.000	1.000	.400	1.000	.625	8.050

TABLE XVIII.

Coefficients of Wh for the Five-Span Bent.

(K of one member varying)

K_A	K_B	K_C	K_D	K_E	K_F	K	A	B	C	D	E	F
1	1	1	1	1	1	0	.0500	.1000	.1000	.1000	.1000	.0500
1	1	1	1	1	1	1/2	.0549	.0990	.0961	.0961	.0990	.0549
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	1	1	1	1	3/2	.0611	.0967	.0921	.0921	.0967	.0611
1	1	1	1	1	1	2	.0632	.0955	.0913	.0913	.0955	.0632
1	1	1	1	1	1	∞	.0833	.0833	.0833	.0833	.0833	.0833
1	1	1	1	1	0	1	.0686	.1151	.1102	.1130	.0930	0
1	1	1	1	1	1/2	1	.0600	.1009	.0966	.0970	.0970	.0485
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	1	1	1	3/2	1	.0576	.0965	.0926	.0926	.0981	.0627
1	1	1	1	1	2	1	.0571	.0958	.0921	.0917	.0981	.0650
1	1	1	1	1	∞	1	.0556	.0933	.0896	.0890	.0991	.0734
1	1	1	1	0	1	1	.0791	.1528	.1297	.1060	0	.0525
1	1	1	1	1/2	1	1	.0630	.1053	.1017	.0965	.0761	.0571
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	1	1	3/2	1	1	.0561	.0942	.0901	.0925	.1080	.0589
1	1	1	1	2	1	1	.0549	.0921	.0879	.0920	.1140	.0594
1	1	1	1	∞	1	1	.0501	.0840	.0797	.0891	.1364	.0606
1	1	1	0	1	1	1	.0766	.1311	.1040	0	.1091	.0796
1	1	1	1/2	1	1	1	.0626	.1052	.0961	.0725	.1001	.0631
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	1	3/2	1	1	1	.0564	.0942	.0926	.1040	.0964	.0561
1	1	1	2	1	1	1	.0553	.0921	.0921	.1100	.0958	.0547
1	1	1	∞	1	1	1	.0513	.0844	.0900	.1321	.0929	.0495

TABLE XIX

Coefficients of Wh for the Five-Span Bent.

(K's of two Columns varying simultaneously)

K_A	K_B	K_C	K_D	K_E	K_F	K	A	B	C	D	E	F
1	1	1	1	0	0	1	.0876	.1468	.1428	.1228	0	0
1	1	1	1	1/2	1/2	1	.0648	.1085	.1042	.0998	.0756	.0474
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	1	1	3/2	3/2	1	.0554	.0930	.0890	.0911	.1083	.0632
1	1	1	1	2	2	1	.0536	.0901	.0860	.0896	.1146	.0661
1	1	1	1	∞	∞	1	.0471	.0791	.0750	.0839	.1385	.0761
1	1	1	0	1	0	1	.0974	.1664	.1310	0	.1055	0
1	1	1	1/2	1	1/2	1	.0646	.1089	.0992	.0751	.0994	.0526
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	1	3/2	1	3/2	1	.0557	.0928	.0917	.1024	.0968	.0603
1	1	1	2	1	2	1	.0541	.0904	.0904	.1080	.0963	.0611
1	1	1	∞	1	∞	1	.0488	.0808	.0864	.1270	.0942	.0629
1	1	0	1	1	0	1	.0966	.1366	0	.1328	.1328	0
1	1	1/2	1	1	1/2	1	.0652	.1035	.0749	.0996	.1049	.0520
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	3/2	1	1	3/2	1	.0554	.0953	.1024	.0915	.0946	.0605
1	1	2	1	1	2	1	.0536	.0940	.1080	.0901	.0926	.0615
1	1	∞	1	1	∞	1	.0477	.0895	.1278	.0855	.0855	.0642
1	0	1	1	1	0	1	.0664	0	.1332	.1659	.1345	0
1	1/2	1	1	1	1/2	1	.0592	.0785	.0997	.1054	.1048	.0524
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	3/2	1	1	1	3/2	1	.0584	.1066	.0915	.0888	.0945	.0605
1	2	1	1	1	2	1	.0581	.1119	.0902	.0860	.0929	.0609
1	∞	1	1	1	∞	1	.0583	.1310	.0856	.0761	.0856	.0634
0	1	1	1	1	0	1	0	.1131	.1370	.1370	.1131	0
1/2	1	1	1	1	1/2	1	.0500	.1000	.1000	.1000	.1000	.0500
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
3/2	1	1	1	1	3/2	1	.0620	.0967	.0912	.0912	.0967	.0620
2	1	1	1	1	2	1	.0636	.0964	.0900	.0900	.0964	.0636
∞	1	1	1	1	∞	1	.0702	.0946	.0851	.0851	.0946	.0702
1	1	1	0	0	1	1	.1030	.1748	.1460	0	0	.0763
1	1	1	1/2	1/2	1	1	.0675	.1133	.1042	.0746	.0781	.0622
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	1	3/2	3/2	1	1	.0543	.0907	.0890	.1027	.1065	.0568
1	1	1	2	2	1	1	.0519	.0865	.0861	.1080	.1120	.0557
1	1	1	∞	∞	1	1	.0431	.0716	.0765	.1264	.1305	.0522
1	1	0	1	0	1	1	.1237	.1688	0	.1287	0	.0786
1	1	1/2	1	1/2	1	1	.0686	.1090	.0791	.0991	.0825	.0616
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	3/2	1	3/2	1	1	.0541	.0930	.1000	.0916	.1040	.0570
1	1	2	1	2	1	1	.0518	.0906	.1032	.0905	.1077	.0563
1	1	∞	1	∞	1	1	.0436	.0817	.1152	.0860	.1200	.0537
1	0	1	1	0	1	1	.0823	0	.1677	.1677	0	.0823
1	1/2	1	1	1/2	1	1	.0621	.0826	.1054	.1054	.0826	.0621
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	3/2	1	1	3/2	1	1	.0570	.1041	.0889	.0889	.1041	.0570
1	2	1	1	2	1	1	.0561	.1074	.0864	.0864	.1074	.0561
1	∞	1	1	∞	1	1	.0530	.1200	.0770	.0770	.1200	.0530

(Continued)

TABLE XIX (Cont'd.)

Coefficients of Wh for the Five-Span Bent.

(K's of two columns varying simultaneously.)

K_A	K_B	K_C	K_D	K_E	K_F	K	A	B	C	D	E	F
1	1	0	0	1	1	1	.1024	.1475	0	0	.1475	.1024
1	1	1/2	1/2	1	1	1	.0679	.1079	.0743	.0743	.1079	.0679
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	3/2	3/2	1	1	1	.0543	.0929	.1029	.1029	.0929	.0543
1	1	2	2	1	1	1	.0518	.0901	.1081	.1081	.0901	.0518
1	1	∞	∞	1	1	1	.0427	.0801	.1270	.1270	.0801	.0427

TABLE XX

Coefficients of Wh for the Five-Span Bent.

(K's of three columns varying simultaneously)

K_A	K_B	K_C	K_D	K_E	K_F	K	A	B	C	D	E	F
1	1	1	0	0	0	1	.1281	.2073	.1706	0	0	0
1	1	1	1/2	1/2	1/2	1	.0695	.1169	.1071	.0772	.0775	.0516
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	1	3/2	3/2	3/2	1	.0535	.0895	.0880	.1011	.1070	.0610
1	1	1	2	2	2	1	.0509	.0847	.0846	.1050	.1126	.0621
1	1	1	∞	∞	∞	1	.0410	.0679	.0726	.1200	.1321	.0661
1	1	0	1	0	0	1	.1441	.1973	0	.1586	0	0
1	1	1/2	1	1/2	1/2	1	.0708	.1120	.0814	.1028	.0819	.0512
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	3/2	1	3/2	3/2	1	.0534	.0918	.0988	.0904	.1048	.0612
1	1	2	1	2	2	1	.0507	.0888	.1012	.0881	.1086	.0629
1	1	∞	1	∞	∞	1	.0413	.0775	.1093	.0815	.1222	.0680
1	0	1	1	0	0	1	.0967	0	.1963	.2070	0	0
1	1/2	1	1	1/2	1/2	1	.0636	.0851	.1085	.1000	.0821	.0517
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	3/2	1	1	3/2	3/2	1	.0561	.1029	.0879	.0879	.1047	.0608
1	2	1	1	2	2	1	.0546	.1052	.0846	.0844	.1084	.0625
1	∞	1	1	∞	∞	1	.0504	.1139	.0733	.0735	.1221	.0672
0	1	1	1	0	0	1	0	.1535	.1883	.1580	0	0
1/2	1	1	1	1/2	1/2	1	.0540	.1079	.1082	.1030	.0780	.0491
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
3/2	1	1	1	3/2	3/2	1	.0596	.0933	.0876	.0900	.1070	.0625
2	1	1	1	2	2	1	.0594	.0908	.0843	.0883	.1128	.0645
∞	1	1	1	∞	∞	1	.0599	.0809	.0719	.0809	.1320	.0733

(Continued)

TABLE XX (Cont'd.)

Coefficients of Wh for the Five-Span Bent.

(K's of three columns varying simultaneously.)

K_A	K_B	K_C	K_D	K_E	K_F	K	A	B	C	D	E	F
1	1	0	0	1	0	1	.1400	.2027	0	0	.1566	0
1	1	1/2	1/2	1	1/2	1	.0699	.1120	.0769	.0773	.1072	.0565
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	3/2	3/2	1	3/2	1	.0536	.0919	.1017	.1011	.0935	.0582
1	1	2	2	1	2	1	.0508	.0886	.1060	.1059	.0908	.0576
1	1	∞	∞	1	∞	1	.0413	.0775	.1250	.1222	.0815	.0544
1	0	1	0	1	0	1	.1176	.0	.1965	0	.1920	0
1	1/2	1	1/2	1	1/2	1	.0639	.0855	.1027	.0822	.1086	.0574
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	3/2	1	3/2	1	3/2	1	.0563	.1030	.0905	.0986	.0934	.0581
1	2	1	2	1	2	1	.0553	.1059	.0889	.1013	.0913	.0576
1	∞	1	∞	1	∞	1	.0519	.1160	.0850	.1108	.0830	.0554
0	1	1	0	1	0	1	0	.1801	.1788	0	.1410	0
1/2	1	1	1/2	1	1/2	1	.0558	.1082	.1031	.0777	.1030	.0544
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
3/2	1	1	3/2	1	3/2	1	.0599	.0935	.0904	.1010	.0958	.0596
2	1	1	2	1	2	1	.0603	.0909	.0883	.1058	.0945	.0603
∞	1	1	∞	1	∞	1	.0619	.0824	.0824	.1220	.0906	.0605
1	0	0	1	1	0	1	.1041	0	0	.2028	.1931	0
1	1/2	1/2	1	1	1/2	1	.0641	.0809	.0772	.1087	.1131	.0563
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	3/2	3/2	1	1	3/2	1	.0560	.1051	.1013	.0880	.0911	.0567
1	2	2	1	1	2	1	.0548	.1097	.1060	.0845	0871	.0581
1	∞	∞	1	1	∞	1	.0504	.1260	.1221	.0732	.0732	.0550
1	1	0	0	0	1	1	.1604	.2272	.0	0	0	.1121
1	1	1/2	1/2	1/2	1	1	.0731	.1171	.0811	.0769	.0848	.0669
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	1	3/2	3/2	3/2	1	1	.0521	.0897	.0988	.1016	.1029	.0549
1	1	2	2	2	1	1	.0487	.0852	.1017	.1060	.1057	.0528
1	1	∞	∞	∞	1	1	.0372	.0696	.1107	.1221	.1147	.0458
1	0	0	1	0	1	1	.1461	.0	0	.2237	0	.1302
1	1/2	1/2	1	1/2	1	1	.0677	.0856	.0822	.1082	.0895	.0668
1	1	1	1	1	1	1	.0582	.0978	.0940	.0940	.0978	.0582
1	3/2	3/2	1	3/2	1	1	.0547	1029	.0989	.0883	.1002	.0549
1	2	2	1	2	1	1	.0528	.1060	.1015	.0851	.1015	.0531
1	∞	∞	1	∞	1	1	.0466	.1165	.1119	.0745	.1040	.0466

IV. Discussion of Results.

Each bent will be taken up separately.

14. One-Span Bent. Figure 10 represents a one-span bent. The left-hand column is designated as Column A and the right-hand column as Column B.

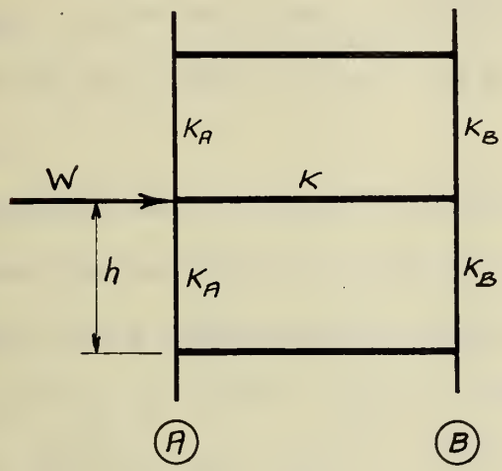


Fig. 10.

From Diagram 1, it is apparent that, no matter what the K of the girder is, if the columns have equal K's the moments at the end of each column is 0.25 Wh.

From Diagram 2, if the K of one column increases the moment in that column increases, but no matter how large K of one column is, if K of the other column equals K of the girder, the moment taken by the larger column does not exceed .27 Wh. If the K of one column is zero the moment in that column is zero and the moment in the other column is .5 Wh.

Hence the moments at the ends of a column may vary from zero to 0.5 Wh.

If the K of either column equals the K of the girder, the moments at the ends of the other column may vary from zero to 0.27 Wh.

Also if the K of either column equals the K of the girder the moments at the ends of the other girder may vary from 0.23 Wh to 0.27 Wh, if the K of the other column varies from one-half the K to twice the K of the girder. This represents a variation of 4% from the mean value of 0.25 Wh. We may conclude then that in a story of a one-span bent the moments at the ends of the two columns may be considered equal, provided the K of either column is not less than one-half the K of the other column and not less than one half the K of the girder.

Diagram 2 shows that, if the K's of one column and the girder are equal, an increase of K of the other column causes a rapid increase in the moment which this column will carry (and a corresponding decrease in the other column) up to the point where the K's of all members are equal. From this point on, the increase in moment is slight. After K has reached an amount twice as large as the K of the other column (and of the girder) the increase in moment is negligible.

We may conclude from this, that for the one-span bent the moment in a column is limited by the K of the girder.

Diagram 3 shows the effect upon the moment in Column A, of the K of column A and also of the K of Column B. Points upon the left-hand line represent the moments in Column A as K of Column A varies, and points upon the right-hand line represent the moments in Column A as K of Column B varies. The line marked 0.5 connects the point on the left-hand line corresponding to K of column A equals 0.5 with the point on the right-hand line corresponding to K of Column B equals 0.5. Similarly for the lines marked 0.0, 1.0, 1.5, 2.0, and ∞. These lines have the peculiarity of

intersecting at very nearly the same point. This diagram also demonstrates that, provided the K of one column is equal to the K of the girder, values of 1/2 K and ∞ for the other column are limits of approximately the same weight.

15. Two-Span Bent. The K's of all girders are equal. Figure 11 represents a two-span bent. Beginning at the left-hand side the columns are designated as Column A, Column B, and Column C.

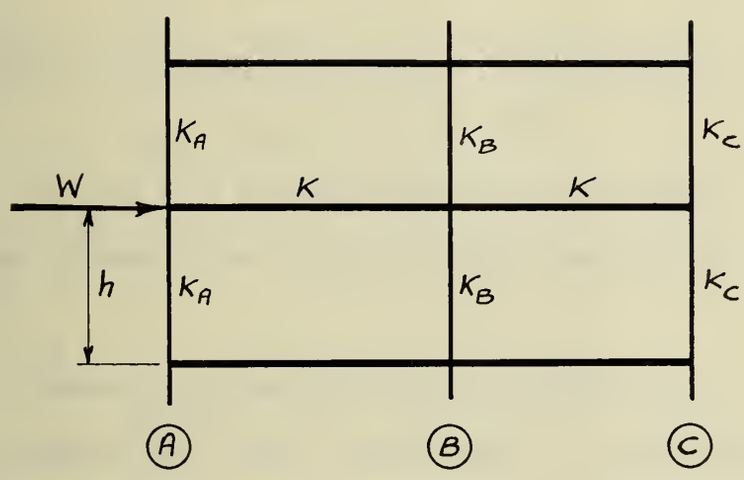


Fig. 11

Tables III and IV show the following peculiarities:

If the K's of all columns in a story are half as large as the K's of the girders, the θ's at the ends of the exterior columns are twice those of the interior column and equal to $\frac{Wh}{36EK_B}$.

If the K of the interior column is twice the K of each of the exterior columns and twice the K of a girder, the θ's at the ends of all columns in the story are equal to each other and equal to $\frac{Wh}{24EK_B}$.

From Tables V and VI it may be seen that:

If the K's of all columns in a story are equal, and the K's of the girders are zero, the moments at the ends of the exterior columns are half those at the ends of the interior column. This result is identical with the one furnished by Method IV.

This is not a rational case. For, from Table III,

$$\frac{Wh}{6ER} = 0.$$

Hence R must equal infinity; and as all

$$\frac{\Theta's}{R} = 1$$

all Θ's must also equal infinity. Expressed in symbols,

$$\Theta_A = \Theta_B = \Theta_C = R = \infty.$$

If the K's of all columns in a story are equal and the K's of the girders are infinity, the moments in all the columns are equal.

Hence, if the K's of all columns in a story are equal, an exterior column will take one-fourth to one-third of the total shear, and the interior column, one-half to one-third of the total shear, depending upon the relation of the K's of the columns to the K's of the girders.

If the K's of the exterior columns in a story are equal to the K's of the girders, and the K of the interior column equals zero, the total shear will be divided equally between the two exterior columns.

If the K's of the exterior columns in a story are half as large as the K of the interior column, the moments at the ends of the exterior columns are half those at the ends of the interior column. This is true no matter what the value of K of the girders may be. This result is identical with the one furnished by Method IV.

Diagrams 5 to 8, inclusive, have the same general form as Diagram 2. They show that the moment in a column is limited by the K of the girder; and that an increase in K of a column beyond twice that of the girder has a negligible effect upon the distribution of moments.

Diagrams 9 and 10 are similar to Diagram 3 described on page 56. Diagram 9 shows the change in moment at the ends of an exterior column and Diagram 10 shows the change in moment at the ends of an interior column, when any <u>one</u> column changes its K.

Again, it may be seen, that the straight lines connecting corresponding ordinates almost intersect at either one or two points, and that values of 1/2 K and ∞ for one column are limits of approximately the same weight, provided the K's of all other columns are equal to the K's of the girders.

The maximum error introduced by neglecting a change of K within the limits of 1/2 and ∞ is about $\pm$ 13%.

It may be further seen, that the moment in an exterior column is influenced mainly by its own K; in a lesser degree by the K of the adjoining interior column; and in an even lesser degree by the K of the other exterior column. However, the differences are but slight. The moment in the interior column is greatly influenced by its own K, while the change of K of an exterior column has a negligible influence of less than $\pm$ 5% between the limits of 1/2 and ∞.

Diagrams 11 to 16 inclusive show the influence of equal simultaneous changes in the K's of <u>two</u> columns in a story.

If the interior and one of the exterior columns vary simultaneously, the moment in this exterior column changes very little (i.e., ± 4%), while the moment in the other exterior column varies appreciably (± 21%). The moment in the interior column varies ± 10%.

An idea of the influence of the K of the girder may be obtained by comparing the following two cases:

(a) $K_A = K_B = 1$ $\quad K_C = 2, \quad K = 1$

(b) $K_C = 1$ $\quad K_A = K_B = 1/2, \quad K = 1.$

This latter case is identical with

(b') $K_A = K_B = 1,$ $\quad K_C = 2, \quad K = 2.$

The coefficients of Wh are:

for (a)	A = .127	B = .229	C = .144
for (b')	A = .128	B = .213	C = .159
Difference:	.001	.016	.015

Considering this influence the above statement should be modified to read:

If the interior and one of the exterior columns vary simultaneously, the moment in this exterior column changes very little (± 4%). The moment in the interior column varies about ±10%. The moment in the other exterior column depends largely on the ratio of the K of that column to that of the girder. If the K of that column is twice that of the girder, the moment will vary ± 8%; if its K is equal to that of the girder, the moment may vary ± 21%.

If the two exterior columns vary simultaneously, all three moments change in about equal proportion (about ± 7%).

16. Three-Span Bent. The K's of all girders are equal.

Figure 12 represents a three-span bent. Beginning at the left-hand side the columns are designated as Column A, Column B, Column C and Column D.

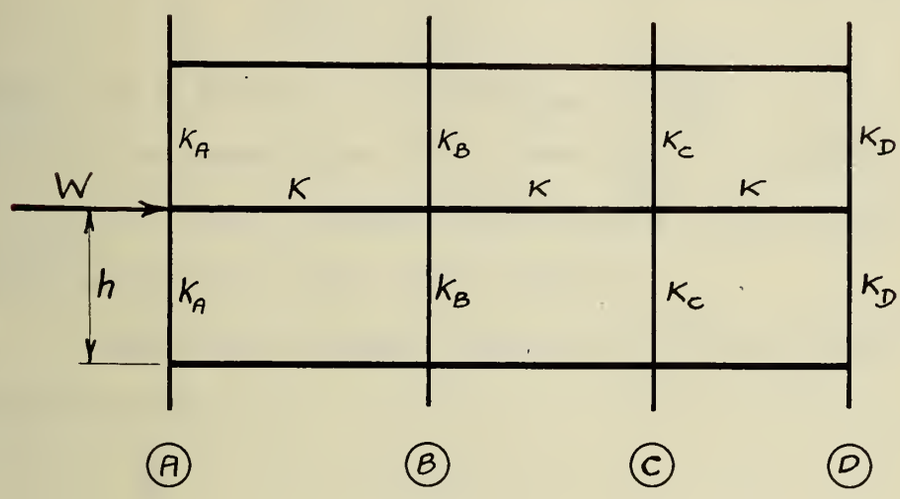


Fig. 12.

Tables VII and VIII show the following peculiarities: If the K's of the columns are equal and the K's of the girders are zero

$$\theta_A = \theta_B = \theta_C = \theta_D = R = \infty.$$

If the K's of the columns are equal, and the K's of the girders are infinity, all θ's = 0. Then,

$$\frac{Wh}{6ER} = 8K$$

and

$$R = \frac{d}{h} = \frac{Wh^2}{48EI}$$

in which I is the moment of inertia of the columns.

This is the smallest R possible under the given wind load hence the smallest horizontal deflection d in a story equals

$$\text{min. } d = \frac{Wh^3}{48EI}$$

$\frac{h^3}{48E}$ may be considered as practically constant from top to bottom of a building, and W changes from max. W at the bottom to zero at the top following a linear law.

W at the mth floor of an n-story building equals

$$W_m = \frac{n - m}{n} \cdot W_{max.}$$

in which, for a strip 1'-0" wide, $W_{max.} = \frac{nhw}{12}$

The denominator 12 is necessary to express "h" in feet.

The total minimum deflection at any story is the sum of all the deflections below that story.

The total minimum deflection Σd at the mth floor of an n-story building is

$$\sum_0^m d = d_1 + d_2 + d_3 + \ldots\ldots\ldots d_m$$

$$\sum_0^m d = \frac{h^3}{48E}\left(\frac{W_1}{I_1} + \frac{W_2}{I_2} + \frac{W_3}{I_3} + \ldots \frac{W_m}{I_m} \right)$$

$$\sum_0^m d = \frac{h^3}{48E} \cdot \frac{W_{max}}{n} \left[\frac{(n-1)}{I_1} + \frac{(n-2)}{I_2} + \frac{(n-3)}{I_3} + \ldots + \frac{(n-m)}{I_m} \right]$$

Assuming an average value I for I_1, I_2, I_3, etc.,

$$\sum_0^m d = \frac{wh^4}{576EI}\left[mn - (1 + 2 + 3 + \ldots\ldots + m)\right]$$

$$\sum_0^m d = \frac{wh^4}{576EI}\left[mn - \frac{m}{2}(m + 1)\right]$$

$$\text{min.} \sum_0^m d = \frac{wh^4 m}{1152EI}\left[2n - m - 1\right]$$

This equation gives the total minimum deflection at the mth floor of a three-span bent, n stories high, under a wind load of w lbs. per sq. ft. on a strip 1'0 wide.

At the 10th floor of a 20-story building, average story height 144", width of bay 16'-0", and for w = 20#/□', average I I = 500 in.^{4}, and E = 30,000,000 #/□", the total minimum deflection to be expected is

$$\min \sum_{o}^{m} d = \frac{20 \times 144^4 \times 10 \times (40 - 10 - 1) \times 16}{1152 \times 30{,}000{,}000 \times 500} = 2.3"$$

As may be seen from Table VII, under average conditions the deflection will be about three times as large as the minimum. In the given example the deflection under average conditions will be

$$\text{average} \sum_{o}^{m} d = 6.9"$$

The minimum deflection at the top of a building is obtained by making

$$m = n,$$

as

$$\sum_{o}^{n} d = \frac{w\ h^4\ n}{1152\ EI}(n - 1)$$

In our example

$$\text{min.} \sum_{o}^{n} d = \frac{20 \times 144^4 \times 20 \times 19}{1152 \times 30{,}000{,}000 \times 500} = 3.0"$$

Under average conditions

$$\text{average} \sum_{o}^{n} d = 9.0"$$

Comparison with Tables I and III shows that the minimum deflection for the one-span bent is twice as large as that for the three-span bent, and the minimum deflection for the two-span bent is four-thirds times that of the three-span bent.

For average conditions the deflections of the one one-span bent as well as of the two-span bent are about three times

as large as their respective minima.

The deflection varies with the fourth power of the average story height.

From Table VIII, a change of K in one exterior column has no influence on the Θ in the other one, which is three bays distant.

If the K's of the exterior columns are half those of the interior columns, the Θ's at the ends of all columns in the story equal $\frac{Wh}{36EK_B}$.

Tables IX and X show that:

If the K's of all columns in a story are equal and the K of the girder is zero, the results are identical with those furnished by Method IV. This case is not rational as has been shown for the two-span bent.

If the K's of all columns in a story are equal and the K of the girder is infinity, the moments at the ends of all columns are equal.

Hence, if the K's of all columns in a story are equal, an exterior column will take one-sixth to one-fourth of the total shear and an interior column one-third to one-fourth of the total shear, depending upon the relation of the K's of the columns to the K's of the girders.

If the K's of the exterior columns in a story are half as large as the K's of the interior columns, the results are identical with those furnished by Method IV.

An idea of the influence of the girder may be obtained from Table X.

For $K_B = K_C = 1$, $K_A = K_D = 2$, $K = 1$

$A = .100$ $B = .150$ $C = .150$ $D = .100$

For $K_B = K_C = 1/2$, $K_A = K_D = 1$, $K = 1$, which is identical with

$K_B = K_C = 1$, $K_A = K_D = 2$, $K = 2$

$A = .111$ $B = .139$ $C = .139$ $D = .111$

Table IX shows also that, if the K's of all columns in a story are equal, the K of the girder may be considered zero, if it is one-tenth that of the columns or less.

The diagrams have the same general form as those for the two-span bent. The statements made there hold here too.

Diagrams 24 to 29 are similar to Diagram 3 described on page 56. Diagram 24 shows the change in moment at the ends of an exterior column, and Diagram 29 shows the change in moments at the ends of an interior column when any one column changes its K.

The maximum error introduced by neglecting a change of K, within the limits of 1/2 and ∞ is about ± 26%.

Hence the relative error is about twice that for the two-span bent.

The moment in an exterior column is influenced mainly by the K of that column and by the K of the interior column two bays distant. The influence of the other exterior column is small and the influence of the other interior column is negligible (± 4%).

The moment in an interior column is influenced mainly by the K of that column; in a much lesser degree by the K of the other interior column and the exterior column furthest from it. The influence of the adjoining exterior column is negligible (± 1%).

Diagrams 25 to 28 inclusive show the influence of a simultaneous change of the K's of two columns upon the moments in an exterior column in a story.

From Diagram 25 it may be seen that a simultaneous and equal change of K of an exterior column and of the distant interior column has a negligible influence upon the moment in that exterior column.

A simultaneous and equal change in an exterior column and the adjoining interior column has a greater influence upon the moment in that exterior column than a simultaneous and equal change in the two exterior columns.

Diagram 26 shows that a simultaneous and equal change of the interior columns has a greater influence on the moment in an exterior column than a simultaneous and equal change in that exterior column and the adjoining interior column.

Diagrams 30 to 33 inclusive show the influence of a simultaneous change of the K's of two columns upon the moments in an interior column in a story.

Diagram 30 shows the influence of a simultaneous and equal change of any column and the exterior column adjoining the interior column under consideration.

Diagram 31 shows the influence of a simultaneous and equal change of any column and the interior column itself.

Diagram 32 shows the influence of a simultaneous and equal change of any column and the other interior column.

Diagram 33 shows the influence of a simultaneous and equal change of any column and the exterior column furthest away from the interior column under consideration.

17. Four-Span Bent. The K's of all girders are equal. Figure 13 represents a four-span bent.

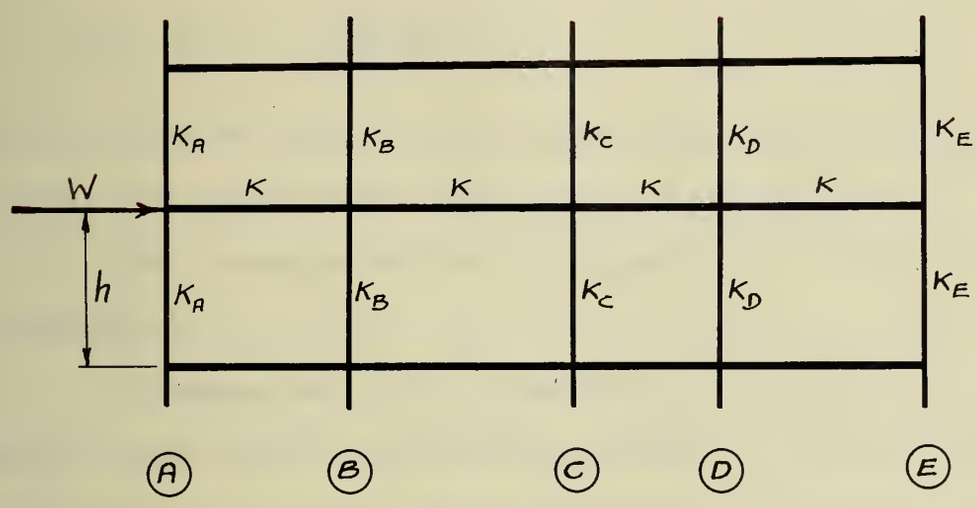


Fig. 13.

Tables XI and XII show the following peculiarities:

If the K's of the columns are equal and the K's of the girders are zero,

$$\theta_A = \theta_B = \theta_C = \theta_D = \theta_E = R = \infty.$$

If the K's of the columns are equal, and the K's of the girders are infinity, all θ's = 0.

Then,

$$\text{min. } R = \frac{Wh^2}{60EI}$$

and

$$\text{min. } d = \frac{Wh^3}{60\ EI}$$

The total minimum deflection at the mth floor of a four-span bent n stories high under a wind load of w lbs. per sq. ft., for a strip 1'-0 wide, is

$$\text{min.} \sum_{o}^{m} d = \frac{wh^4 m}{1440\ E\ I}\left[2n - m - 1\right]$$

in which I is an average moment of inertia of the columns.

The total minimum deflection at the top of a four-span bent

$$\text{min.} \sum_{o}^{n} d = \frac{wh^4 n}{1440 EI}\left[n - 1\right]$$

is four-tenths of that of a one-span bent, six-tenths that of a two-span bent and eight-tenths that of a three-span bent.

The average deflection appears to be about three times the minimum.

A change of K in a column has no influence on the θ in another column which is three bays distant. The same thing is true if more than one column change their K's simultaneously.

If the K of the innermost column is twice that of the other columns and of the girder, the θ's at the ends of this column are equal to those at the ends of the exterior columns.

If the K of the exterior columns is half that of the interior columns, the θ's at the ends of all columns in the story equal $\frac{Wh}{48E\,K_B}$.

Tables XIII and XIV show that

If the K's of all columns in a story are equal and the K's of the girders are zero, the results are identical with those furnished by Method IV. This case is not rational as has been shown for the two-span bent.

If the K's of all columns in a story are equal and the K's of the girders are infinity, all the moments are equal.

Hence, if the K's of all columns in a story are equal, an

exterior column will take one-eighth to one-fifth of the total shear, and an interior column, one-fourth to one-fifth of the total shear, depending upon the relation of the K's of the columns to the K's of the girders.

If the K's of the exterior columns in a story are half as large as the K's of the interior columns, the results are identical with those furnished by Method IV.

The diagrams for this bent have the same general form as for the preceding bents. The statements made for the two-span bent hold here too.

Diagram 44 shows the change in moment at the ends of an exterior column, and Diagrams 50 and 56 show the change in moments at the ends of an interior column if any <u>one</u> column changes its K.

The maximum error introduced by neglecting a change of K, within the limits of 1/2 and ∞ is ± 25%, which is about the same as for the three-span bent.

The moment in an exterior column is influenced by the K of that column and in the same degree by the K of any interior column except the one adjoining, which has no influence at all. The influence of the other exterior column is a little over ± 5%.

The moment in an interior column next to an exterior column is not influenced by the latter.

The moment in the innermost column is influenced mainly by the K of that column.

The remaining diagrams from 45 to 61 inclusive show the influence of a simultaneous and equal change of the K's of <u>two</u> columns upon the moments in any column.

18. Five-Span Bent. The K's of all girders are equal. Figure 14 represents a five-span bent.

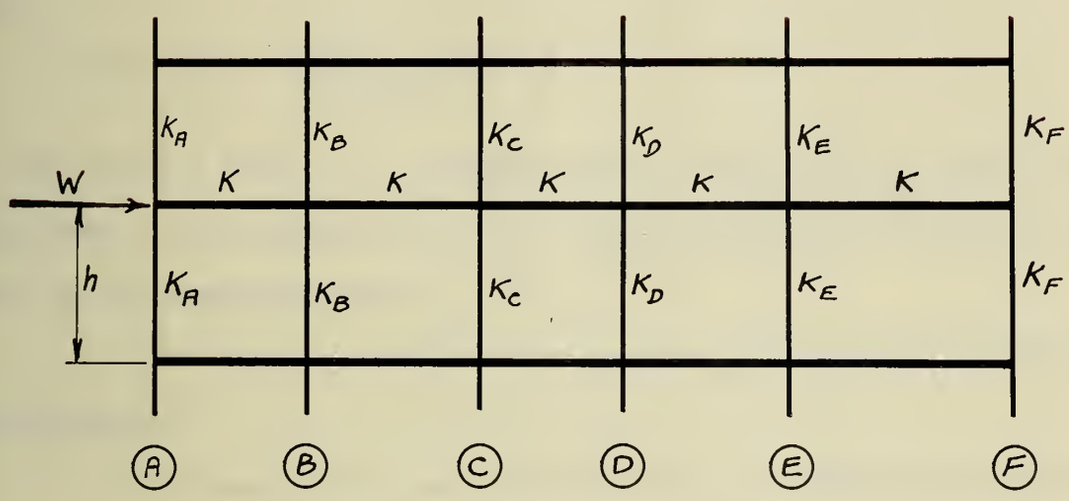


Fig. 14.

Tables XV, XVI and XVII show the following peculiarities:

If the K's of the columns are equal and the K's of the girders are zero,

$$\theta_A = \theta_B = \theta_C = \theta_D = \theta_E = \theta_F = R = \infty .$$

If the K's of the columns are equal, and the K's of the girders are infinity, all θ's = 0.

Then,

$$\text{min. } R = \frac{Wh^2}{72\ EI}$$

and

$$\text{min. } d = \frac{Wh^3}{72\ EI}$$

The total minimum deflection at the mth floor of a five-

span bent, n stories high, under a wind load of w lbs. per sq. ft., is

$$\text{min.} \sum_{o}^{m} d = \frac{wh^4 m}{1728\ EI} \left[2n - m - 1\right]$$

in which I is an average moment of inertia of the columns.

The total minimum deflection at the top of a five-span bent,

$$\text{min.} \sum_{o}^{n} d = \frac{wh^4 n}{1728\ EI} \left[n - 1\right],$$

is two-sixths that of a one-span bent, three-sixths that of a two-span bent, four-sixths that of a three-span bent and five-sixths that of a four-span bent.

The average deflection appears to be about three times the minimum.

A change of K in a column has no influence on the θ's at the end of a column three bays distant.

If the K's of the exterior columns are half those of the interior columns, the θ's at the ends of all columns in the story equal $\frac{Wh}{60E\,K_B}$.

Table XVII also shows a peculiar relation between the slopes at the ends of the columns for the following two cases:

If $K_A = K_D = K_E = K = 1$, and $K_B = K_C = K_F = 2$, $\theta_A = \theta_B = \theta_C$;

and

If $K_A = K_B = K_F = K = 1$, and $K_C = K_D = K_E = 2$, $\theta_D = \theta_E = \theta_F$.

Tables XVIII, XIX and XX show that if the K's of all columns in a story are equal and the K's of the girders are zero, the results are identical with those furnished by Method IV. That this is not a rational case has been shown for the two-span bent.

If the K's of all columns in a story are equal and the K's of the girders are infinity, all the moments in the story are equal.

Hence, if the K's of all columns in a story are equal, an exterior column will take one-tenth to one-sixth of the total shear, and an interior column, one-fifth to one-sixth of the total shear, depending upon the relation of the K's of the columns to the K's of the girders.

If the K's of the exterior columns in a story are half as large as the K's of the interior columns, the results are identical with those furnished by Method IV.

The diagrams for this bent have the same general form and characteristics as those for the preceding bents.

Diagram 85 shows the change in moment at the ends of an exterior column, and Diagrams 107 and 129 show the change in moments at the ends of an interior column, if any <u>one</u> column changes its K.

The maximum error introduced by neglecting a change of K within the limits of 1/2 and ∞ is ± 26%, which is about the same as for the four-span bent and for the three-span bent.

The moment in an exterior column is influenced mainly by the K of that column and in a lesser degree by the K of every interior column. Of the interior columns, the one immediately adjoining has the least influence. The influence of the other exterior column is negligible (± 3%).

The moment in an interior column next to an exterior column is influenced mainly by the K of that column (± 39%). It is not appreciably influenced by the adjoining exterior column (± 1%). The adjoining interior and the distant exterior column have about

equal influence (± 5%). Also the remaining two interior columns have about equal influence (± 14%).

The moment in one of the innermost columns is influenced mainly by the K of that column. The two adjoining interior columns and the two exterior columns have about equal influence (± 5%). The remaining interior column has an influence (± 15%) which is appreciable.

Diagrams 87 to 91 inclusive show the influence of a simultaneous and equal change of the K's of two columns upon the moments in an exterior column.

Diagrams 92 to 106 inclusive show the influence of a simultaneous and equal change of the K's of three columns upon the moments in an exterior column.

Diagrams 108 to 113 and 130 to 135 inclusive show the influence of a simultaneous and equal change of the K's of two columns upon the moments in any interior column.

Diagrams 114 to 128 and 136 to 150 inclusive show the influence of a simultaneous and equal change of the K's of three columns upon the moments in any interior column.

All of the above diagrams may be considered as influence lines for the distribution of shear.

19. A Discussion of Method IV. Method IV is by far the shortest of the approximate methods proposed, and gives good results. It is based on the following assumptions:

(1) The point of contra-flexure of each column is at mid-height of the story.

(2) The point of contra-flexure of each girder is at its mid-length.

(3) The shears on the interior columns are equal, and the shear on each exterior column is equal to one-half of the shear on an interior column.

The agreement with the Approximate Slope Deflection Method is perfect when the K's of the exterior columns are half those of the interior columns.

The agreement is perfect also when all columns in a story have equal K's and the K's of the girders are zero. This, however, is not a rational case.

A decrease in the K's of the girders reduces the shear taken by the exterior columns and consequently increases the shear in the interior columns.

Diagrams for the moments as calculated by the Approximate Slope- Deflection Method (full lines) and by Method IV (broken lines) have been plotted.

for the one-span bent in Diagrams 1 and 2.
" " two-span " " " 4 to 8 inclusive
" " three-span" " " 17 to 23 "
" " four-span " " " 34 to 43 "
" " five-span " " " 62 to 84 "

If all columns in a story have the same K, the maximum error in the moment as determined by Method IV will occur when the K's of the girders are infinity. Table XXI gives this maximum error in per cent of the value found by the Approximate Slope Deflection Method.

TABLE XXI.

Maximum Error by Method IV, K's of All Columns Equal.

K's of Girders Equal Infinity.

Bent	Exterior Columns (on unsafe side)	Interior Columns (on safe side)
One-Span	zero	(100%)
Two-Span	25%	50%
Three-Span	34%	34%
Four-Span	37%	25%
Five-Span	40%	20%

If all columns in a story have equal K's and the K's of the girders are twice those of the columns, the errors by Method IV are given in Table XXII.

TABLE XXII.

Errors by Method IV. K's of All Columns Equal. K's of Girders are Twice Those of the Columns.

Bent	Exterior Columns (unsafe side)	Interior Columns (safe side)
Two-span	11%	12%
Three-span	17%	12%
Four-span	19%	12%
Five-span	20%	10%

If all columns and girders in a story have equal K's the errors by Method IV are given in Table XXIII.

TABLE XXIII

Errors by Method IV if All Columns and Girders Have Equal K's.

Bent	Exterior Columns (unsafe side)	Interior Columns (safe side)
Two-Span	6%	7%
Three-Span	12%	7%
Four-Span	13%	4%
Five-Span	14%	2%

If all columns in a story have equal K's and the K's of the girders are half those of the columns, the errors by Method IV are given in Table XXIV.

TABLE XXIV

Errors by Method IV. K's of All Columns Equal. K's of Girders are Half Those of the Columns.

Bent	Exterior Columns (unsafe side)	Interior Columns. (safe side)
Two-Span	4%	4%
Three-Span	7%	4%
Four-Span	8%	2%
Five-Span	9%	1%

For a story in which the K's of all columns are equal and in which the K's of all girders are equal, Method IV will furnish values which err on the safe side for the interior columns. The

error will be less, the smaller the ratio of the K of the girders is to that of the columns, and the greater the number of spans. The error will be reasonable (maximum 7% for the two-span bent) as long as this ratio does not exceed "1". However, for the exterior columns Method IV errs considerably on the unsafe side. The error increases with the number of spans and with a decrease in the ratio of the K of the girders to the K of the columns.

It has been shown that Method IV agrees with the Approximate Slope Deflection Method when the K's of the interior columns are equal and twice as large as the K's of the exterior columns, no matter what the K's of the girders are. Only, the latter have to be the same for all spans in the story.

20. A Discussion of Method I. Method I is another method giving good results. It is more involved than Method IV as it considers the K's of the girders to some extent.

It will be well to repeat briefly the assumptions which characterize Method I. They are:

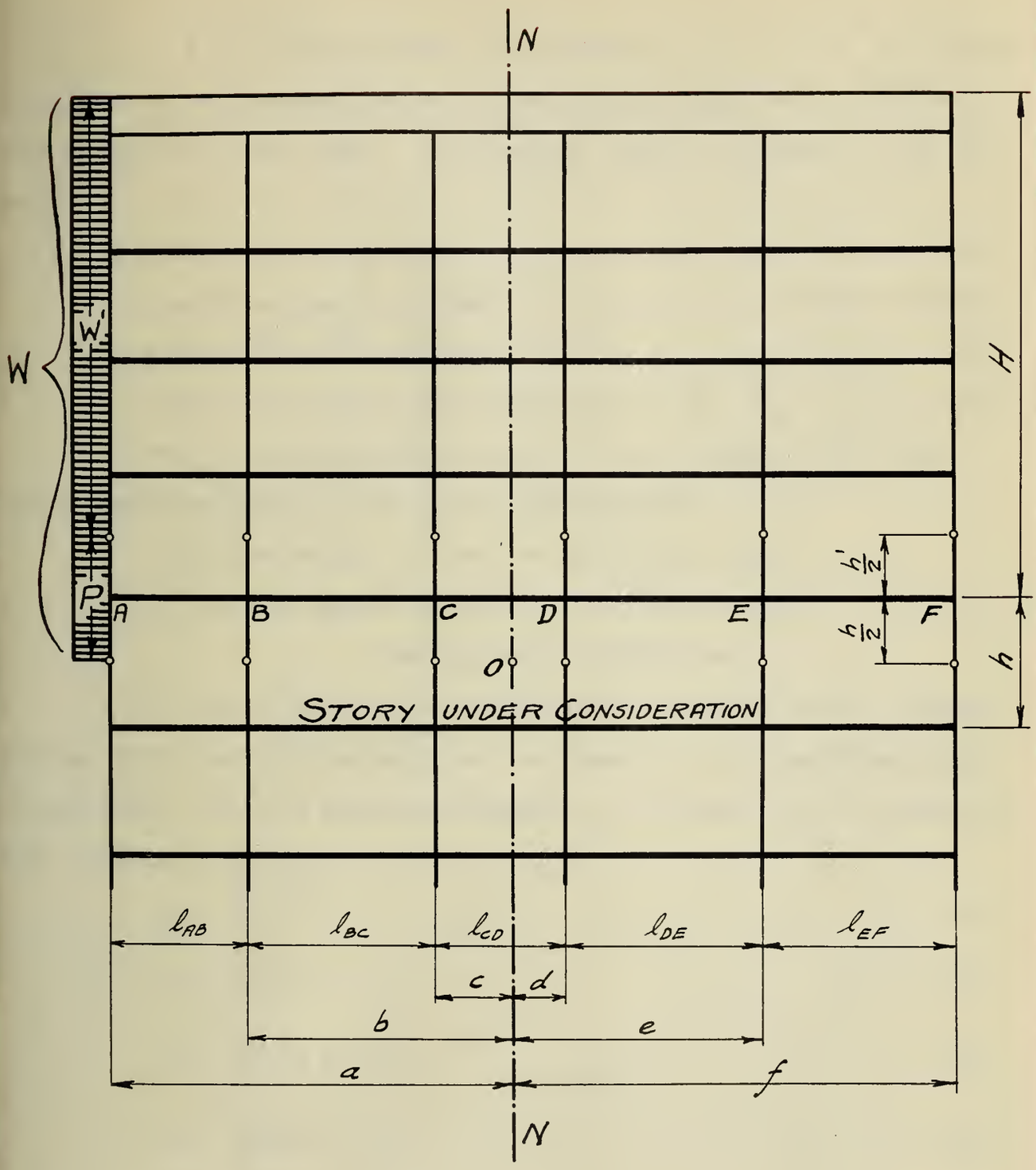
(1) All columns in any given story have the same sectional area and the same moment of inertia.

(2) The point of contraflexure of each column is at mid-height of the story.

(3) The point of contraflexure of each girder is at its midlength.

(4) The direct stress in a column is directly proportional to the distance from the column to the neutral axis of the bent. That is, $D_A : D_B : D_C : \ldots$ etc. $= a : b : c : \ldots$ etc.

In order to arrive at a general conclusion let Figure 15 represent an irregular five-span bent. Let N-N be the neutral axis;



$$P = W - W' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

Fig. 15.

a, b, c, d, e, f, the distances of Columns A, B, C, D, E, F, respectively from this neutral axis, to be counted positive to the left and negative to the right. The neutral axis is defined as one for which

$$a + b + c + d + e + f = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Let N represent the sum of the squares of these distances:

$$N = a^2 + b^2 + c^2 + d^2 + e^2 + f^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

The span lengths are designated as ℓ_{AB}, ℓ_{BC}, ℓ_{CD}, ℓ_{DE} and ℓ_{EF}.

"H" is the distance in feet from the top of the story under consideration to the top of the building.

h is the height in feet of the story, and

h′ is the height in feet of the story above.

Derivation of Equations.

In order to calculate the direct stresses in the columns of the story under consideration, we equate the overturning moment about point O to the resisting moment, and making use of assumption (4), obtain

$$D_A = \frac{W}{4N}(2H + h).a \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$D_B = \frac{W}{4N}(2H + h).b \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$D_C = \frac{W}{4N}(2H + h).c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

$$D_D = \frac{W}{4N}(2H + h).d \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$D_E = \frac{W}{4N}(2H + h).e \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

$$D_F = \frac{W}{4N}(2H + h).f \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

To find the direct stresses in the columns in the story above, we substitute W′ for W, and -h′ for +h.

$$D'_A = \frac{W}{4N}(2H - h').a \quad . \; . \; . \;

The moments at the top of Column A

$$M_A = X.\frac{h}{2} = Wh.\ \frac{a.\ \ell_{AB}}{8N}\ .\ \frac{4H + h - h'}{2H + h - h'}, \quad \ldots\ldots\ldots\ldots (10)$$

Fig. 17 represents joint B.

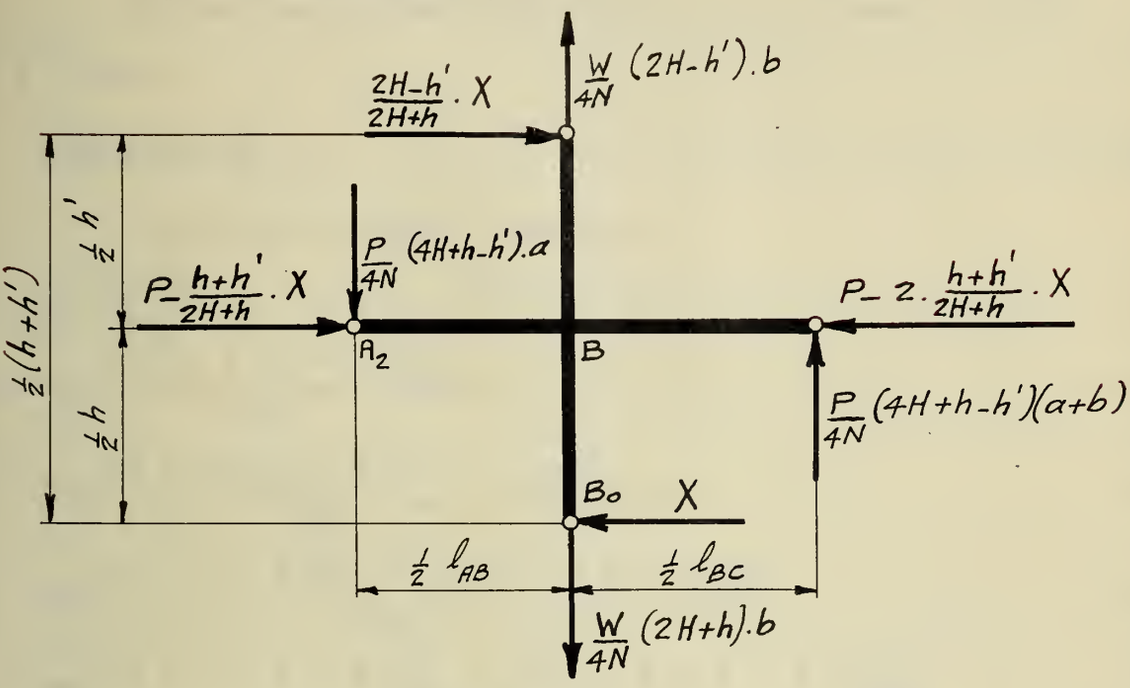


Fig. 17.

Equating moments about point B_o to zero and solving for X, we obtain

$$X = W.\ \frac{a.\ \ell_{AB} + (b + a).\ell_{BC}}{4N}\ .\ \frac{4H + h - h'}{2H + h - h'}$$

and

$$M_B = Wh.\ \frac{a.\ \ell_{AB} + (b + a).\ell_{BC}}{8N}\ .\ \frac{4H + h - h'}{2H + h - h'} \quad \ldots\ldots\ldots (11)$$

Progressing from joint to joint, we obtain

$$M_C = Wh.\ \frac{(b + a).\ell_{BC} + (c + d + a).\ell_{CD}}{8N}\ .\ \frac{4H + h - h'}{2H + h - h} \quad \ldots\ldots (12)$$

$$M_D = Wh.\ \frac{(c + b + a).\ell_{CD} + (d + c + b + a).\ell_{DE}}{8N}\ \frac{4H + h - h'}{2H + h - h'} \quad \ldots\ldots (13)$$

$$M_E = Wh.\ \frac{(d + c + b + a).\ell_{DE} + (e + d + c + b + a).\ell_{EF}}{8N}\ .\ \frac{4H + h - h'}{2H + h - h'} \quad \ldots (14)$$

$$M_F = Wh.\ \frac{(e + d + c + b + a).\ell_{EF}}{8N}\ \frac{4H + h - h'}{2H + h - h'} \quad \ldots\ldots\ldots (15)$$

in which the quantities d, e, and f are negative, being measured to the <u>right</u> of the neutral axis.

If the two consecutive stories are of equal height, $h = h'$, and

$$\frac{4H + h - h'}{2H + h - h'} = 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

The above formulas reduce to

$$M_A = Wh.\ \frac{a.\ell_{AB}}{4N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (I)$$

$$M_B = Wh.\ \frac{a.\ell_{AB} + (b + a).\ell_{BC}}{4N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (II)$$

$$M_C = Wh.\ \frac{(b + a).\ell_{BC} + (c + b + a).\ell_{CD}}{4N} \quad \ldots\ldots\ldots\ldots\ldots \quad (III)$$

$$M_D = Wh.\ \frac{(c + b + a).\ell_{CD} + (d + c + b + a).\ell_{EF}}{4N} \quad \ldots\ldots\ldots\ldots \quad (IV)$$

$$M_E = Wh.\ \frac{(d + c + b + a).\ell_{DE} + (e + d + c + b + a).\ell_{EF}}{4N} \quad \ldots\ldots \quad (V)$$

$$M_F = Wh.\ \frac{(e + d + c + b + a).\ell_{EF}}{4N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (VI)$$

The important equation

$$a + b + c + d + e + f + \ldots = 0 \quad \ldots\ldots\ldots\ldots \quad (1)$$

may be made use of in combination with these formulas. Thus, equations (VI), (V) and (IV) may be written

$$M_F = -\ Wh.\ \frac{f.\ell_{EF}}{4N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (VI')$$

$$M_E = -\ Wh.\ \frac{f.\ell_{EF} + (e + f).\ell_{DE}}{4N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (V')$$

$$M_D = -\ Wh.\ \frac{(e + f).\ell_{DE} + (d + e + f).\ell_{CD}}{4N} \quad \ldots\ldots\ldots\ldots \quad (IV')$$

showing an analogy with equations (I), (II), and (III).

With other words, it is simpler to calculate the moments to one side of the neutral axis independent of those to the other side.

To check the results we use the relation

$$\Sigma M = M_A + M_B + M_C + M_D + M_E + M_F = \frac{Wh}{2}$$

$$\Sigma M = \frac{Wh}{2N}\left[a.\ell_{AB}+(b+a)\ell_{BC} + (c+b+a)\ell_{CD} + (d+c+b+a)\ell_{DE} +(e+d+c+b+a)\ell_{EF}\right]$$

$$\Sigma M = \frac{Wh}{2N}\left[a(\ell_{AB}+\ell_{BC}+ \ldots +\ell_{EF})+b(\ell_{BC}+\ell_{CD}+\ell_{DE}+\ell_{EF})+c(\ell_{CD}+\ell_{DE}+\ell_{EF})+ + d(\ell_{DE}+\ell_{EF})+ e\ell_{EF}\right]$$

$$\Sigma M = \frac{Wh}{2N}\left[a(a - f)+ b(b - f) + c(c - f)+ d(d - f) + e(e - f)\right]$$

$$\Sigma M = \frac{Wh}{2N}\left[(a^2 + b^2 + c^2 + d^2 + e^2 - f(a + b + c + d + e)\right]$$

Substituting from equation (1)

$$a + b + c + d + e = -f$$

$$\Sigma M = \frac{Wh}{2N}\left[a^2 + b^2 + c^2 + d^2 + e^2 + f^2\right] = \frac{Wh}{2N} \cdot N$$

$$\Sigma M = \frac{Wh}{2}$$

If the two consecutive stories have not exactly the same height, the error due to considering them equal is not serious in most cases. That is, the quantity $\frac{4H + h - h'}{2H + h - h'}$, does not vary much from 2.00. Near the ground where large variations in story-height are likely to occur, H will be large as compared with h. Near the top of a building the wind load is comparatively small and the error has no serious effect. For example, say H_1 = 200'-0 (near the ground) H_2 = 25'-0 (near the top), h = 10'-0, h′ = 16'-0.

$$\frac{4H_1 + h - h'}{2H_1 + h - h'} = 2.02, \text{ error} = 1.0\% \text{ near the ground.}$$

$$\frac{4H_2 + h - h'}{2H_2 + h - h'}, = 2.14, \text{ error} = 7.0\% \text{ near the top.}$$

If h and h′ are reversed, the error is smaller and on the side of safety.

The moments in the girders are also obtained from Figures 2 and 3. For the joints to be in equilibrium

$$M_{AB} = M_{BA} = \frac{P}{8N}.\ (4H + h - h').a.\ell_{AB} \quad \ldots\ldots\ldots\ldots\ldots \quad (19)$$

The moment to the right of Column B

$$M_{BC} = M_{CB} = \frac{P}{8N}(4H + h - h').\ (b + a).\ell_{BC} \quad \ldots\ldots\ldots\ldots \quad (20)$$

Similarly,

$$M_{CD} = \frac{P}{8N}(4H + h - h')\ (c + b + a).\ell_{CD} \quad \ldots\ldots\ldots\ldots\ldots \quad (21)$$

etc.

If $h = h'$, or if $h - h'$ is very small as compared with 4H, the equations reduce to

$$M_{AB} = P.H.\ \frac{a.\ell_{AB}}{2N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (VII)$$

$$M_{BC} = P.H.\ \frac{(b + a).\ell_{BC}}{2N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (VIII)$$

$$M_{CD} = P.H.\ \frac{(c + b + a).\ell_{CD}}{2N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (IX)$$

$$M_{DE} = P.H.\ \frac{(d + c + b + a).\ell_{DE}}{2N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (X)$$

$$M_{EF} = P.H.\ \frac{(e + d + c + b + a).\ell_{EF}}{2N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (XI)$$

P.H. is only slightly larger than W. The difference is $\frac{P}{2}$ h. The formulas deduced for the five-span bent hold without modification for a bent with any number of spans.

If column F is not an exterior but an interior column, equation (VI) will read

$$M_F = Wh.\ \frac{(e + d + c + b + a)\ell_{EF} + (f + e + d + c + b + a)\ell_{FG}}{4N} \quad \ldots \quad (XII)$$

Equations (I) to (XII), whose structure is easily committed to memory, suffice to solve for bents up to ten spans. It is easy to write as many more as may be required.

The advantages claimed for these formulas are: to allow

the moments at any joint to be calculated independent from any other; to show at a glance which items influence the moments at any joint and to what extent; and to furnish results more rapidly than Method I.

For the special case when all spans have equal lengths l let

t = total number of columns in a story.

r = number of any column counting from left to right as shown in Figure 18.

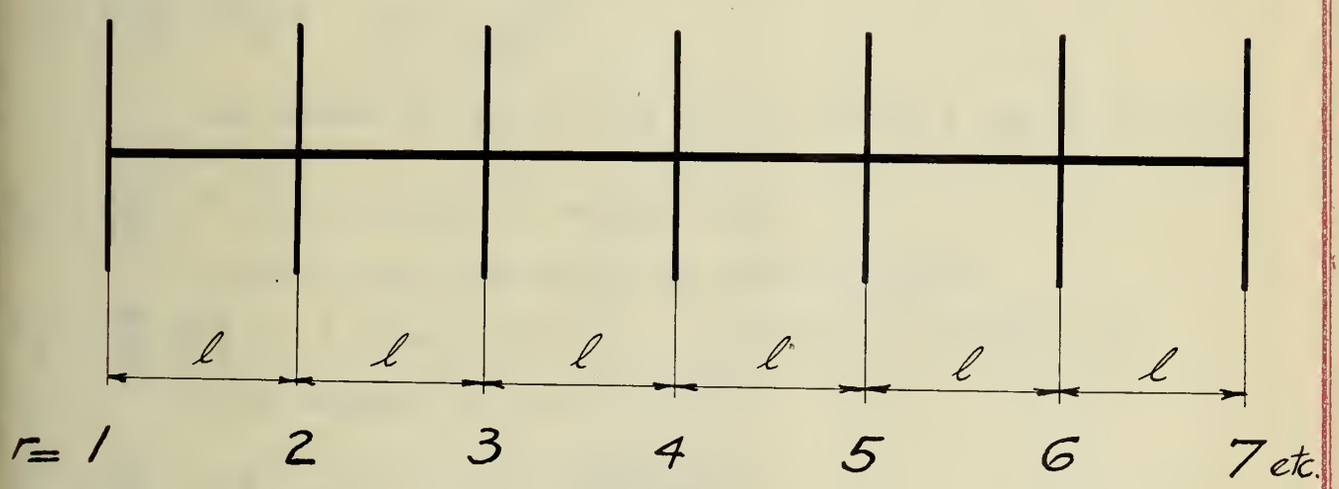


Fig. 18.

In such a case, the distances from the neutral axis (the center line) are

for Column 1 ..... $a_1 = \frac{t - 1}{2} . l$

" " 2 ..... $a_2 = \frac{t - 3}{2} . l$

" " 3 ..... $a_3 = \frac{t - 5}{2} . l$

for any " r ..... $a_r = \frac{t - (2r - 1)}{2} . l$

for the last" t .....$a_t = \frac{-t + 1}{2} . l$

Substituting the values of the a's, in the expression for N, gives

$$N = \frac{\ell^2}{4}\left[(t-1)^2+(t-3)^2+(t-5)^2+ \ldots+(t-2r+1)^2+\ldots+(-t+1)^2\right]$$

This reduces to the form *)

$$4N = \frac{\ell^2}{3} \cdot t(t-1)(t+1)$$

The direct stress in any column r

$$D_r = W. \frac{(2H + h).a_r}{4N}$$

Introducing values for a_r and N,

$$D_r = \frac{3}{2}.W. \frac{2H + h}{\ell} \cdot \frac{t + 1 - 2r}{t(t-1)(t+1)}$$

The moment M_r at the end of any column r may be written

$$M_r = \frac{Wh}{4N} \cdot \ell \cdot \left[2(a_1+a_2+a_3+ \ldots+a_{r-1}) + a_r\right]$$

Substituting the values of the a's, gives

$$M_r = \frac{Wh}{4N} \cdot \ell^2\left\{\left[t-1\right] + \left[t-3\right] + \left[t-5\right]+ \ldots+ \left[t-(2r-3)\right]+ \left[\frac{t-(2r-1)}{2}\right]\right\}$$

This reduces to the form *)

$$M_r = \frac{Wh}{4N} \cdot \ell^2\left\{t(r-1)-(r-1)^2 + \frac{t-(2r-1)}{2}\right\} \text{ or}$$

$$M_r = Wh. \frac{3}{2} \cdot \frac{(t + 1 - r)(2r - 1) - r}{t(t - 1)(t + 1)} \quad \ldots\ldots\ldots\ldots \quad \text{(XIII)}$$

This equation gives the moment at the top of the rth column from the left no matter whether t is an even or an odd number.

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*)Use has been made of the following series (see American Civil Engineers' Pocket Book):

$$1 + 2 + 3 + 4 + \ldots + (r-1) + r = \frac{r}{2}(r+1).$$

$$1 + 3 + 5 + 7 + \ldots + (2r-3)+(2r-1) = r^2$$

$$1^2 + 2^2 + 3^2 + 4^2 + \ldots + (r-1)^2 + r^2 = \frac{1}{6}r(r+1)(2r+1)$$

from which has been deduced

$$1^2 + 3^2 + 5^2+ \ldots + (2r-1)^2 = \frac{r}{3}(2r-1)(2r+1)$$

The moment at the end of the girder to the right of the rth column is

$$M_{r,\ r+1} = P.H.\ \frac{3r(t - r)}{t(t-1)(t+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (XIV)$$

Example: Five-span bent with all spans equal

$t = 6$

Column 1 .... $r = 1$ ..... $M_1 = \frac{Wh}{140}(6x1-1) = \frac{Wh}{140} \cdot 5$

" 2 .... $r = 2$ ..... $M_2 = \frac{Wh}{140}(5x3-2) = \frac{Wh}{140} \times 13$

" 3 .... $r = 3$ ..... $M_3 = \frac{Wh}{140}(4x5-3) = \frac{Wh}{140} \times 17$

" 4 .... $r = 4$ ..... $M_4 = \frac{Wh}{140}(3x7-4) = \frac{Wh}{140} \times 17$

" 5 .... $r = 5$ ..... $M_5 = \frac{Wh}{140}(2x9-5) = \frac{Wh}{140} \times 13$

" 6 .... $r = 6$ ..... $M_6 = \frac{Wh}{140}(1x11-6) = \frac{Wh}{140} \times 5$

Girder 1-2 ...$r = 1$ $M_{12} = \frac{P.H}{70}.(1 \times 5) = \frac{P.H}{70}.\ 5$

" 2-3 ...$r = 2$ $M_{23} = \frac{P.H}{70}.(2 \times 4) = \frac{P.H}{70}.8$

" 3-4 ...$r = 3$ $M_{34} = \frac{P.H}{70}.(3 \times 3) = \frac{P.H}{70}.9$

" 4-5 ...$r = 4$ $M_{45} = \frac{P.H}{70}.(4 \times 2) = \frac{P.H}{70}.8$

" 5-6 ...$r = 5$ $M_{56} = \frac{P.H}{70}.(5 \times 1) = \frac{P.H}{70}.5$

<u>Summary of Article 20.</u> The direct stresses in the columns of a story are

$$D_A = W.\frac{(2H + h).a}{4N}$$

$$D_B = W.\frac{(2H + h).b}{4N}$$

$$D_C = W.\frac{(2H + h).c}{4N}$$

etc.

If two consecutive stories are of equal height, the moments at the tops of the columns in a story are:

$$M_A = Wh.\ \frac{a.\ell_{AB}}{4N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (I)$$

$$M_B = Wh.\ \frac{a.\ell_{AB} + (b + a)\ell_{BC}}{4N} \quad \ldots\ldots\ldots\ldots \quad (II)$$

$$M_C = Wh.\ \frac{(b + a)\ell_{BC} + (c + b + a)\ell_{CD}}{4N} \quad \ldots\ldots\ldots \quad (III)$$

$$M_D = Wh.\ \frac{(c + b + a)\ell_{CD} + (d + c + b + a)\ell_{DE}}{4N} \quad \ldots\ldots \quad (IV)$$

etc.

The moments at the ends of the girders in a story are:

$$M_{AB} = P.H.\ \frac{a.\ell_{AB}}{2N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (VII)$$

$$M_{BC} = P.H.\ \frac{(b + a).\ell_{BC}}{2N} \quad \ldots\ldots\ldots\ldots\ldots \quad (VIII)$$

$$M_{CD} = P.H.\ \frac{(c + b + a).\ell_{CD}}{2N} \quad \ldots\ldots\ldots\ldots\ldots \quad (IX)$$

$$M_{DE} = P.H.\ \frac{(d + c + b + a).\ell_{DE}}{2N} \quad \ldots\ldots\ldots\ldots \quad (X)$$

etc.

If two consecutive stories are of equal height and all spans have equal lengths, the direct stress in the rth column from the left

$$D_r = \frac{3}{2}.W.\ \frac{2H + h}{\ell}\ .\ \frac{t + 1 - 2r}{t(t-1)(t+1)}$$

the moment at the top of the rth column from the left

$$M_r = Wh.\ \frac{3}{2}\ .\ \frac{(t + 1 - r)(2r - 1) - r}{t(t-1)(t+1)} \quad \ldots\ldots\ldots\ldots \quad (XIII)$$

and the moment at the ends of the girder to the right of the rth column

$$M_{r,r+1} = P.H.\ \frac{3r(t-r)}{t(t-1)(t+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (XIV)$$

Moments in bents having all spans equal are given in the following table. The story under consideration and the story above have equal height.

Moments Due to Wind Loads in Steel Frames of Office Buildings.
Spans Equal and Two Consecutive Story Heights Equal.

No. of Spans in Bent	Moments in Columns in terms of Wh.						Moments in Girders in terms of P.H				
	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Span 1	Span 2	Span 3	Span 4	Span 5
1	$\frac{1}{4}$	$\frac{1}{4}$					$\frac{1}{2}$				
2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$				$\frac{1}{4}$	$\frac{1}{4}$			
3	$\frac{3}{40}$	$\frac{7}{40}$	$\frac{7}{40}$	$\frac{3}{40}$			$\frac{3}{20}$	$\frac{4}{20}$	$\frac{3}{20}$		
4	$\frac{2}{40}$	$\frac{5}{40}$	$\frac{6}{40}$	$\frac{5}{40}$	$\frac{2}{40}$		$\frac{2}{20}$	$\frac{3}{20}$	$\frac{3}{20}$	$\frac{2}{20}$	
5	$\frac{5}{140}$	$\frac{13}{140}$	$\frac{17}{140}$	$\frac{17}{140}$	$\frac{13}{140}$	$\frac{5}{140}$	$\frac{5}{70}$	$\frac{8}{70}$	$\frac{9}{70}$	$\frac{8}{70}$	$\frac{5}{70}$

The formulas for the moments in this summary hold strictly only if the story under consideration and the story above have equal height. If they are of unequal height, the moments of the columns are to be multiplied by the factor $\frac{4H + h - h'}{4H + 2h - 2h'}$, and the moments of the girders by the factor $\frac{4H + h - h'}{4H}$.

These factors approach unity as $(h - h')$ approaches zero and for practical purposes may be considered equal to unity. With other words, the error, due to considering two consecutive stories of unequal height as being of equal height, is not serious.

Problem: To find a symmetrical five-span bent for which Method I and Method IV give identical moments

(a) in column A (see Figure 19)

(b) in column B.

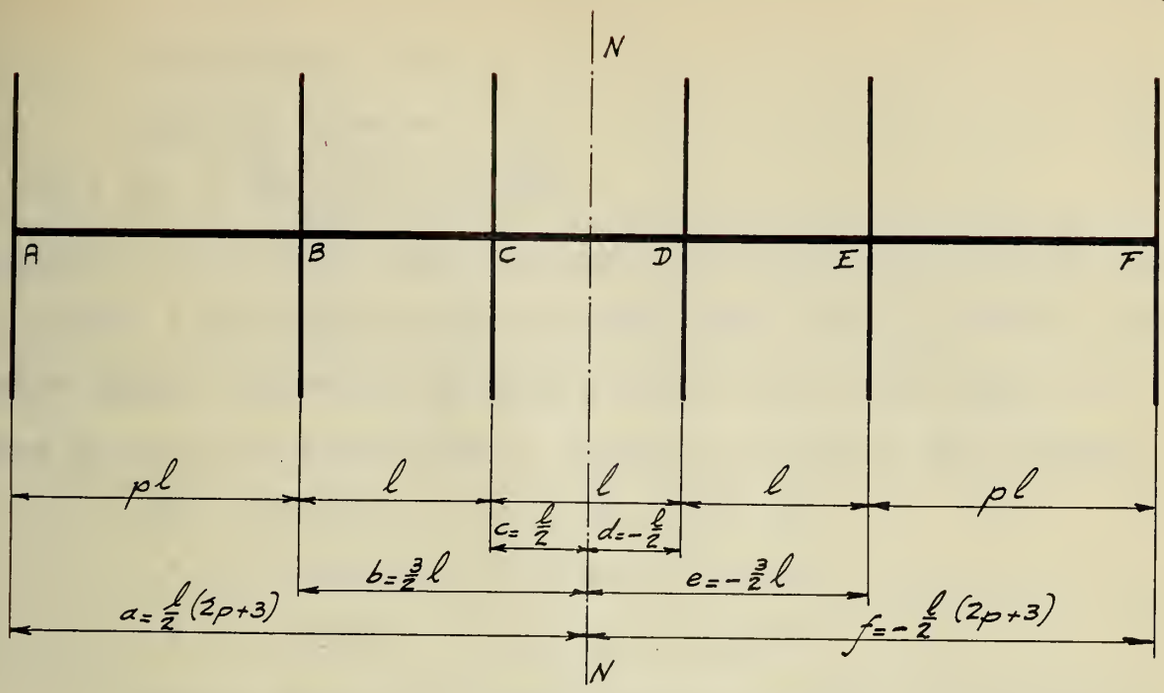

Fig. 19.

Solution for (a):

$$N = \frac{l^2}{2}(19 + 12p + 4p^2)$$

$$M_A = \frac{Wh}{2} \cdot \frac{3p + 2p^2}{38 + 24p + 8p^2} \quad \text{. . . . . Method I}$$

$$M_A = \frac{Wh}{2} \cdot \frac{1}{10} \quad \text{. . . . . Method IV}$$

hence

$$\frac{3p + 2p^2}{38 + 24p + 8p^2} = \frac{1}{10}$$

From this equation p is obtained as p = 1.55.

If the three interior spans of a five-span bent have equal lengths and the two exterior spans are about one and one-half times as long as the interior spans, Methods I and IV will furnish similar results for the exterior columns.

If the exterior spans are longer, Method I will furnish higher values for M_A than Method IV; if they are shorter it will furnish smaller values.

Solution for (b):

Applying formula

$$M_B = Wh. \frac{a.\ell_{AB} + (b + a).\ell_{BC}}{4N}$$

we obtain p = 1.76 for identical moments by Methods I and IV. If p is larger (the exterior spans longer than 1.76), Method I will furnish higher values for M_B than Method IV and vice versa. A change in p has no great effect, however, as may be seen below:

for p = 1.00 ..... M_B = .093 Wh

" p = 1.76 ..... M_B = .100 Wh

" p = 2.00 ..... M_B = .101 Wh

Various other solutions of this problem are possible, of course.

Method I has not been compared with the Approximate Slope Deflection Method, as in this investigation the K of all girders were kept equal to each other. It is possible that Method I will give better results than Method IV where the K of the girders in a story vary, provided the moments of inertia are the same for all girders.

21. Conclusions.

(1) The moment in a column depends on the number of spans in the bent and increases with the ratio of the K of that column to the K's of the other columns. The moment is limited however by the K's of the girders.

(2) A decrease in the K's of the girders reduces the shear taken by the exterior columns and increases the shear taken by the interior columns.

(3) The agreement between Method IV and the Approximate Slope-Deflection Method is perfect when the K's of the interior

columnsare equal to each other and twice as large as the K's of the exterior columns. The K's of the girders have no bearing provided they are equal. The agreement between Method IV and the Approximate Slope-Deflection Method is perfect also when the K's of all columns are equal and the K's of all girders are zero. If the K's of all columns in a story are equal, the K's of the girders may be considered zero as far as the effect upon the moment is concerned if they are one-tenth those of the columns or less.

(4). For a story in which the K's of all columns are equal and in which the K's of all girders are equal, Method IV will furnish values which err on the side of safety for the interior columns. The error will be less, the smaller the ratio of the K's of the girders is to that of the columns, and the greater the number of spans. The error will be reasonable (max. 7% for the two-span bent) as long as the ratio of the K's of the girders tothe K's of the columns does not exceed "1". However, for the exterior columns Method IV errs considerably on the side of danger. The error increases with the number of spans and with a decrease in the ratio of the K of the girders to the K of the columns. When the K's of the girders equal the K's of the columns the maximum error is 14%.

(5). In order to err on the side of safety, exterior columns should be designed as though the shear were divided equally among all columns, and the interior columns, according to Method IV.

(6) A change of K in a column has no effect upon the θ's at the ends of a column three bays distant.

(7) The moments at the ends of the columns in a story of a one-span bent may be considered equal, provided the K of either column is not less than one-half the K of the other column and not

less than one-half the K of the girder.

(8) The horizontal deflection of a building due to wind load varies directly with the fourth power of the average story height.

Under equal conditions, the minimum deflections of corresponding stories in a one-span bent to those in a two-span bent, to those in a three-span bent to those in a four-span bent to those in a five-span bent are as 3.0 to 2.0 to 1.5 to 1.2 to 1.0.

Under "equal conditions" is understood the same wind pressure, the same average story height, the same number of stories and the same average moment of inertia and modulus of elasticity for all columns.

(9) The formulas for Method I presented in article 20 offer certain advantages as compared with the method of considering the forces which put each joint in equilibrium.

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ONE-SPAN BENT

Diagrams 1 to 3, inclusive.

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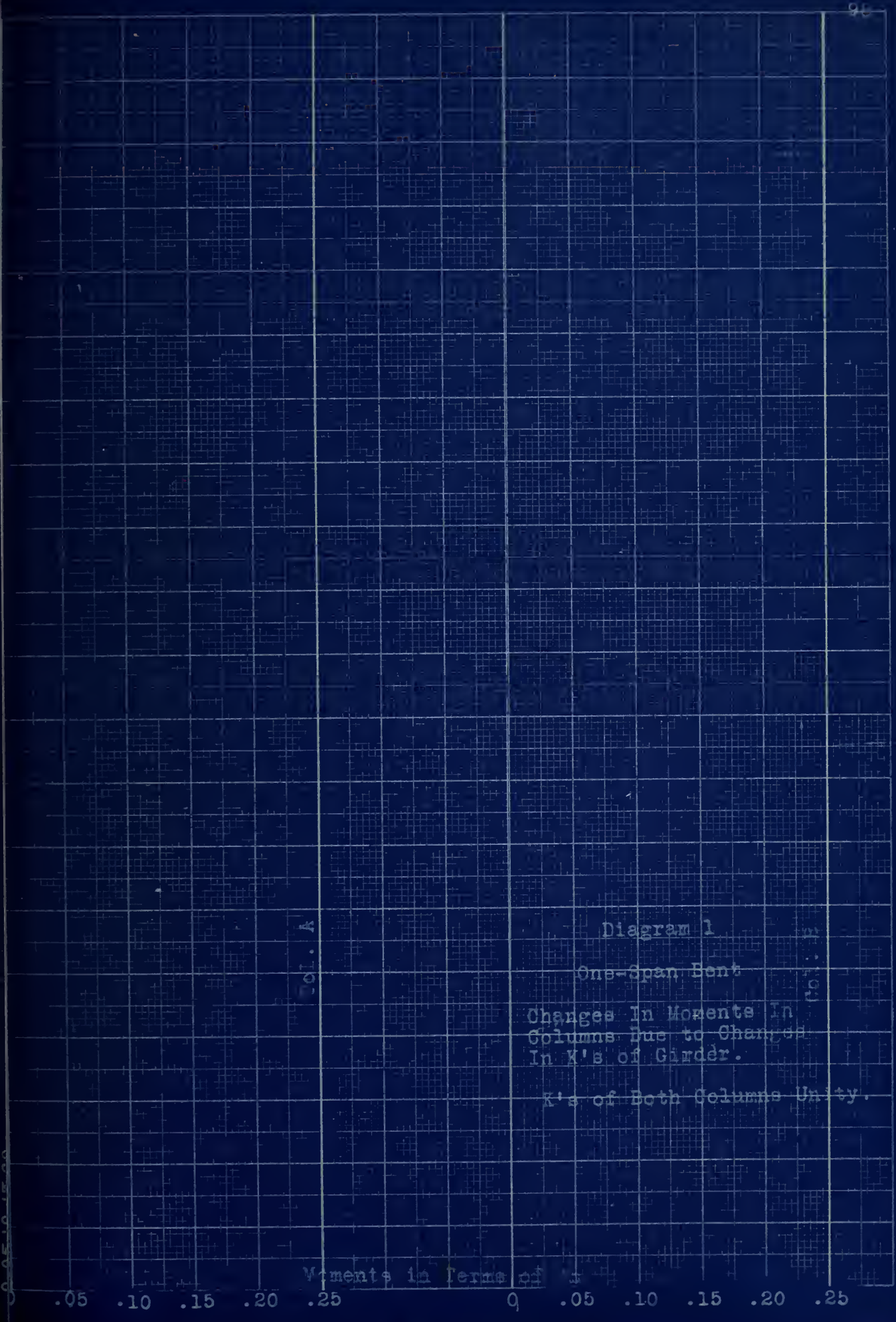

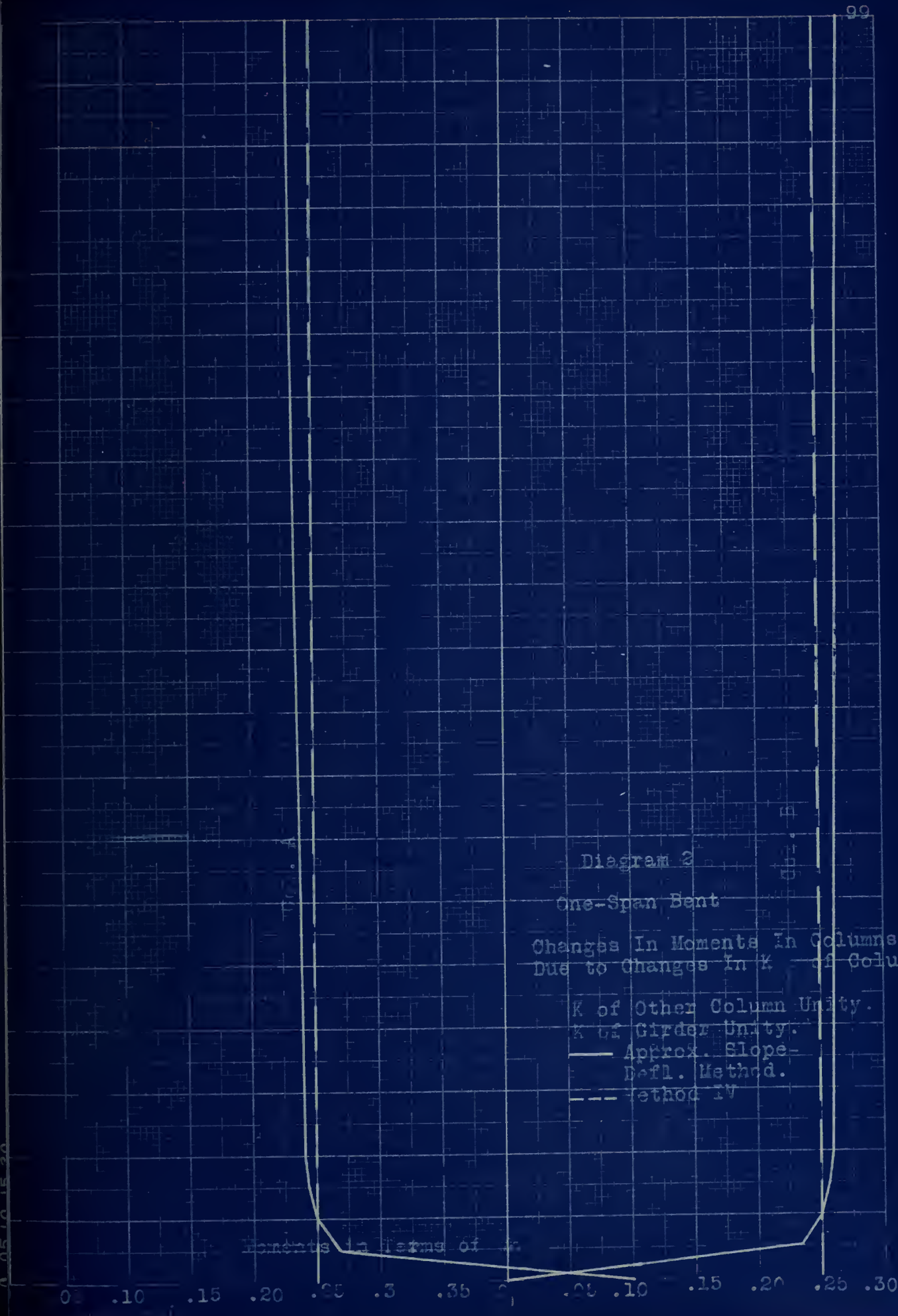
Diagram 2
One-Span Bent
Changes In Moments In Columns
Due to Changes In K of Column B
K of Other Column Unity.
K of Girder Unity.
Approx. Slope-
Defl. Method.
Method IV
Moments in Terms of K
.10
.15
.20
.3
.35
.10
.15
.20
.25
.30

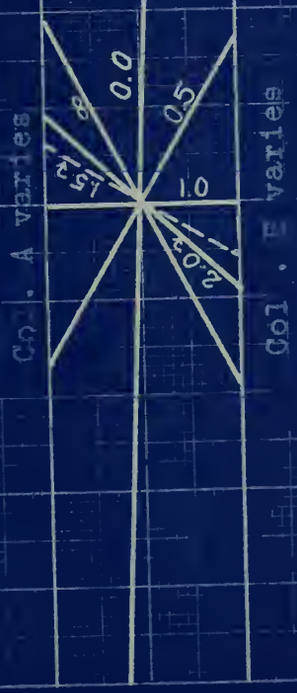

Diagram 3.

One -Span Bent

Moments in Column A.

K of one Column Changes.
K of other Column Unity.
K of Girder Unity.
Numbers on Curves indicate magnitudes of K for member which varies.

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TWO-SPAN BENT

Diagrams 4 to 16, inclusive.

Diagram 4.
Two-Span Bent.
Changes In Moments In Columns
Due To Changes in K's Of Girders.
K's of All Columns Unity
K's of All Girders Equal.
—— Approx. Slope-Defl. Method
- - - Method IV.
Col. A
Col. B
Col. C
Moments in Terms of Wh.
.05 .10 .15 0 .05 .10 .15 .20 .25 0 .05 .10 .15 .20

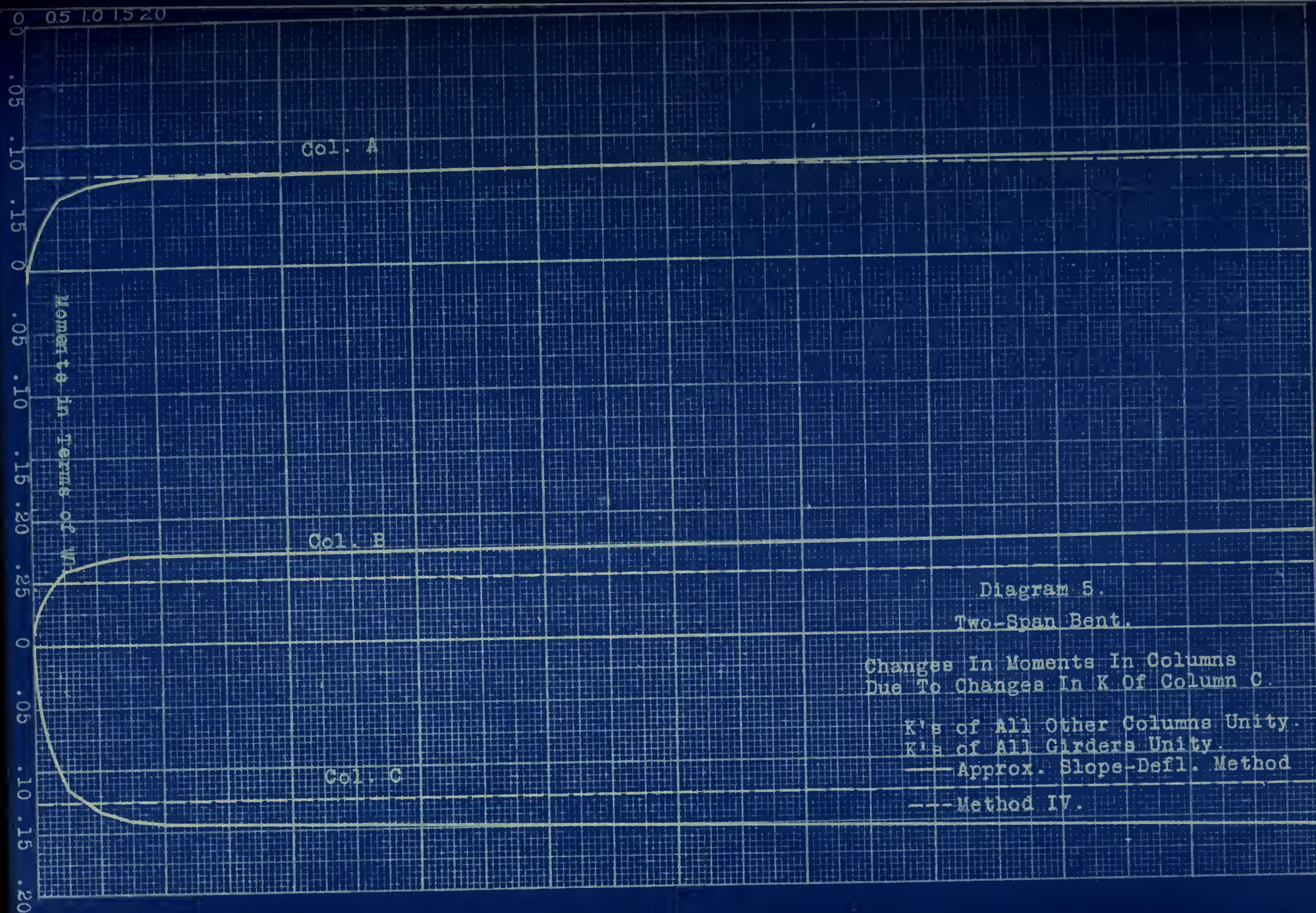


Diagram 5.

Two-Span Bent.

Changes In Moments In Columns
Due To Changes In K Of Column C.

K's of All Other Columns Unity.
K's of All Girders Unity.
——Approx. Slope-Defl. Method
- - -Method IV.

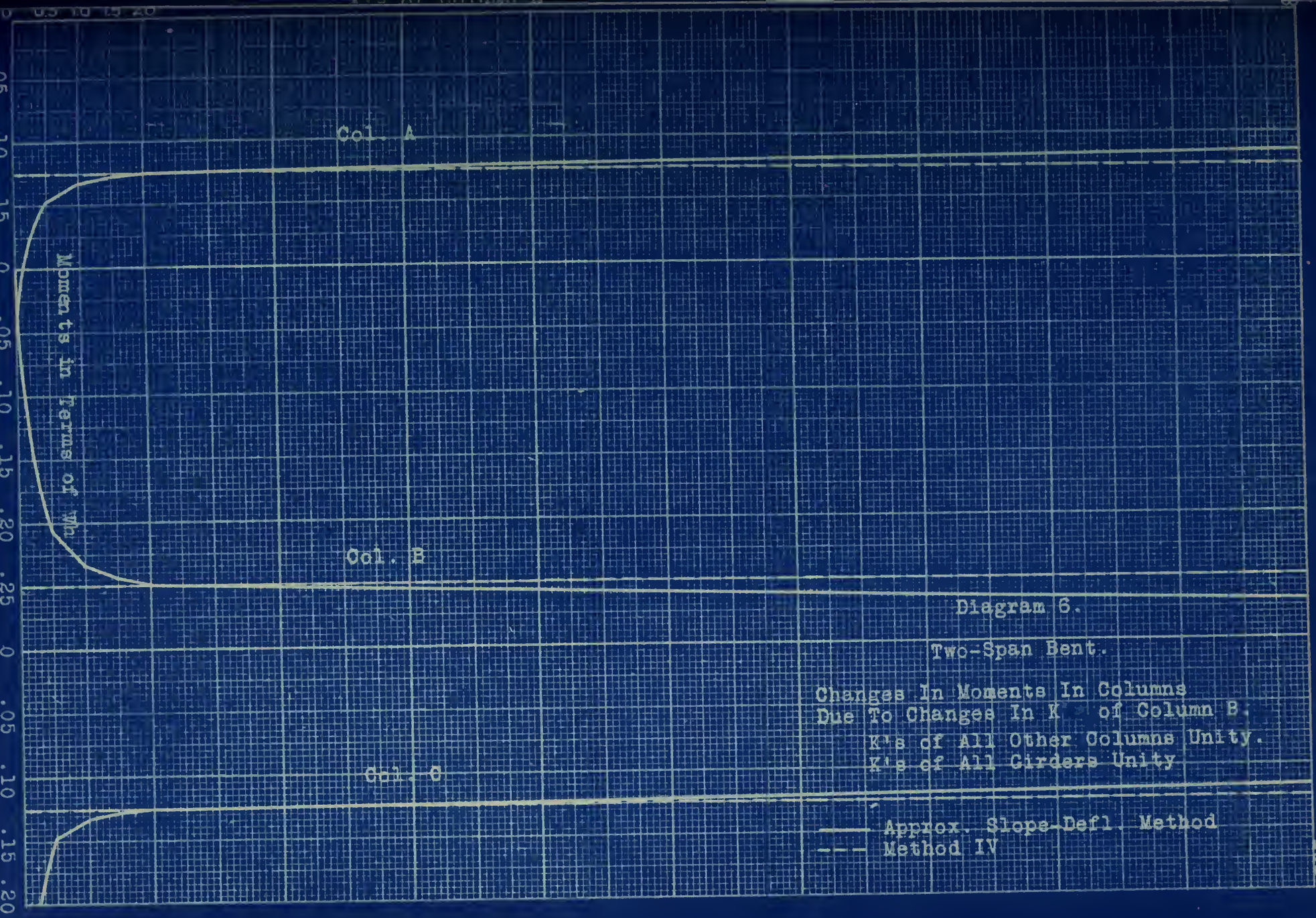


Diagram 6.

Two-Span Bent.

Changes In Moments In Columns
Due To Changes In K of Column B.
K's of All Other Columns Unity.
K's of All Girders Unity

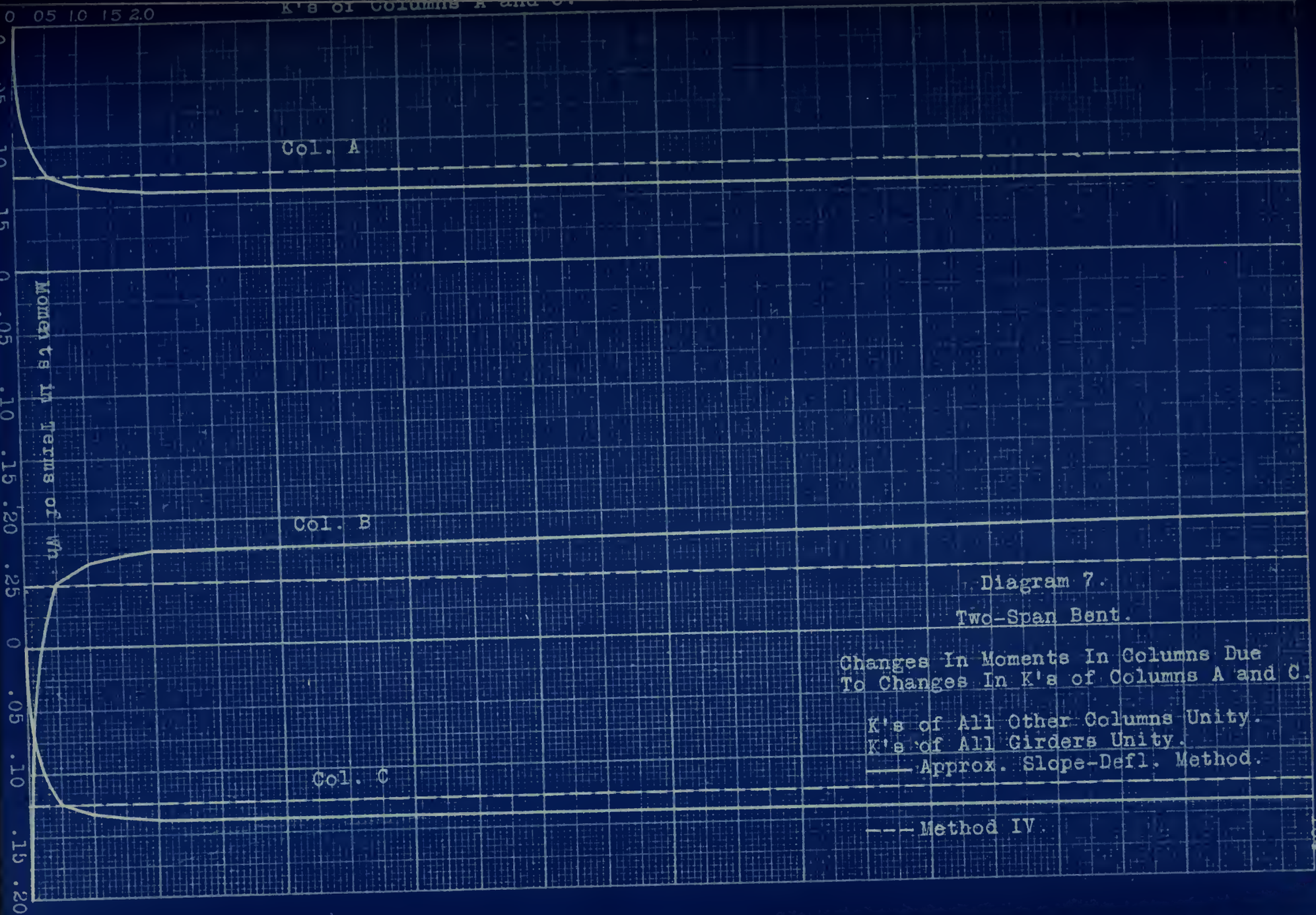


Diagram 7.

Two-Span Bent.

Changes In Moments In Columns Due To Changes In K's of Columns A and C.

K's of All Other Columns Unity.
K's of All Girders Unity.
——— Approx. Slope-Defl. Method.

- - - Method IV.

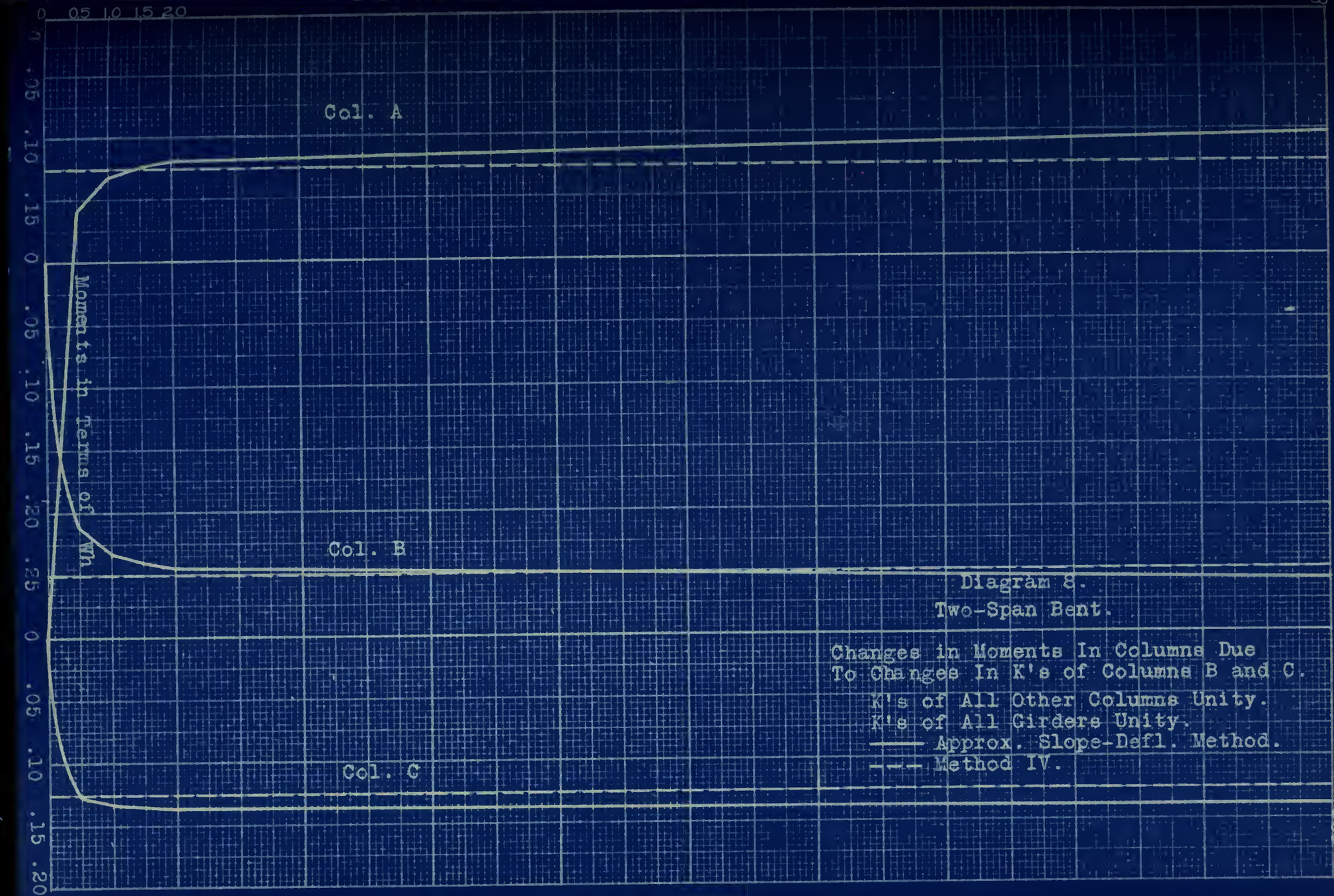

0 0.5 1.0 1.5 2.0
.05 .10 .15 0 .05 .10 .15 .20 .25 0 .05 .10 .15 .20
Moments in Terms of Wh
Col. A
Col. B
Col. C
Diagram 8.
Two-Span Bent.
Changes in Moments In Columns Due
To Changes In K's of Columns B and C.
K's of All Other Columns Unity.
K's of All Girders Unity.
—— Approx. Slope-Defl. Method.
- - - Method IV.

Diagram 9.

Diagram 10.

Two-Span Bent.
Moments in Column A (Diagram 9)
And
Moments in Column B (Diagram 10)

K of One Column Changes
K of Other Columns Unity
K of Girder Unity
Numbers on Curves indicate magnitude of K for member which varies.

Diagram 11

Diagram 12

Two-Span Bent
Moments in Col. A (Diagram 11)
And
Moments in Col. B (Diagram 12)
K's of Colum A and One Other Colum Change Simultaneously.
K's of All Other Columns Unity
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K for Members which vary.

Diagram 13

Diagram 14

Two-Span Bent

Moments In Columns A (Diagram 13)
And
Moments In Column B (Diagram 14)

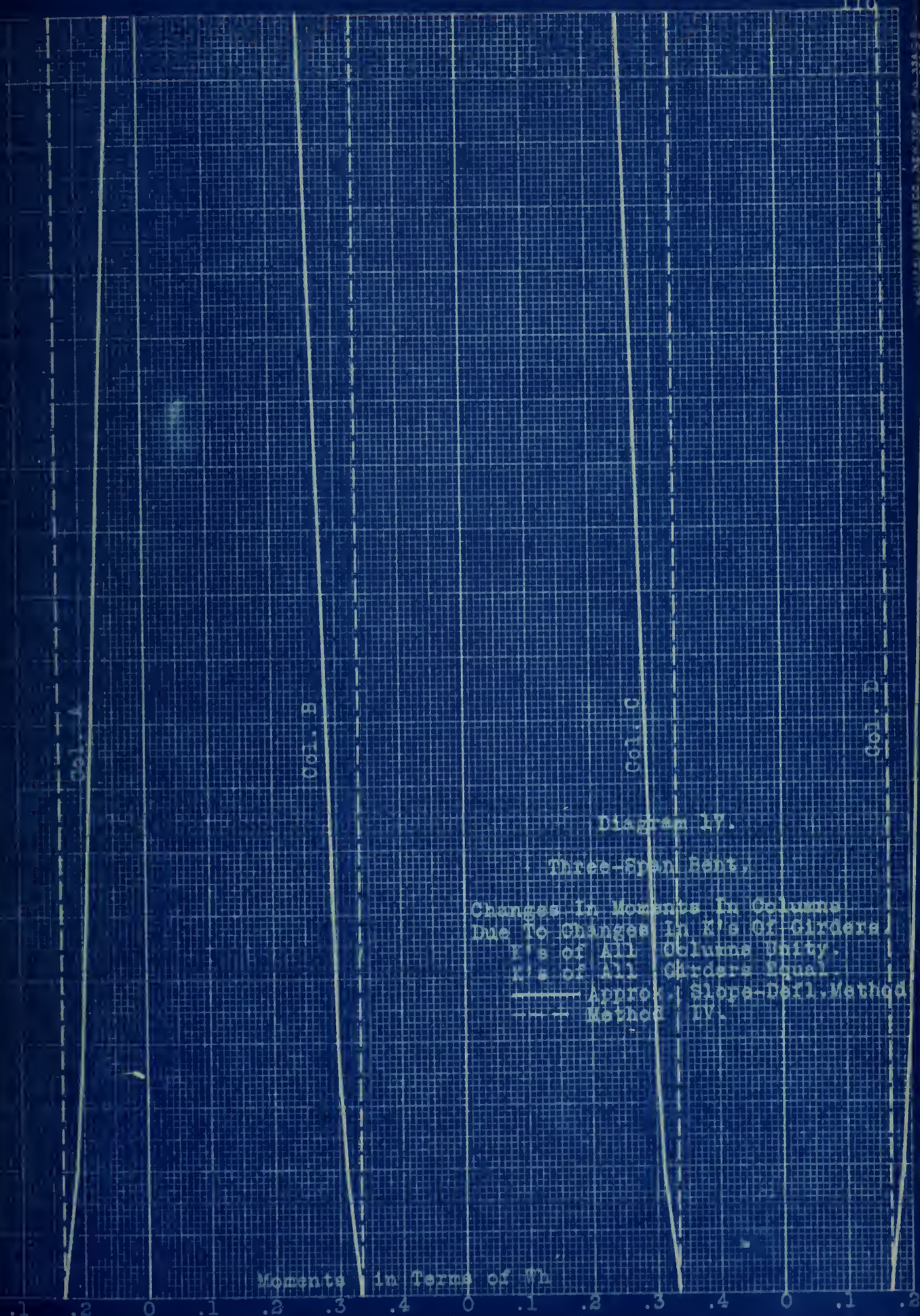
K's of Column B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitude of K for members which vary.

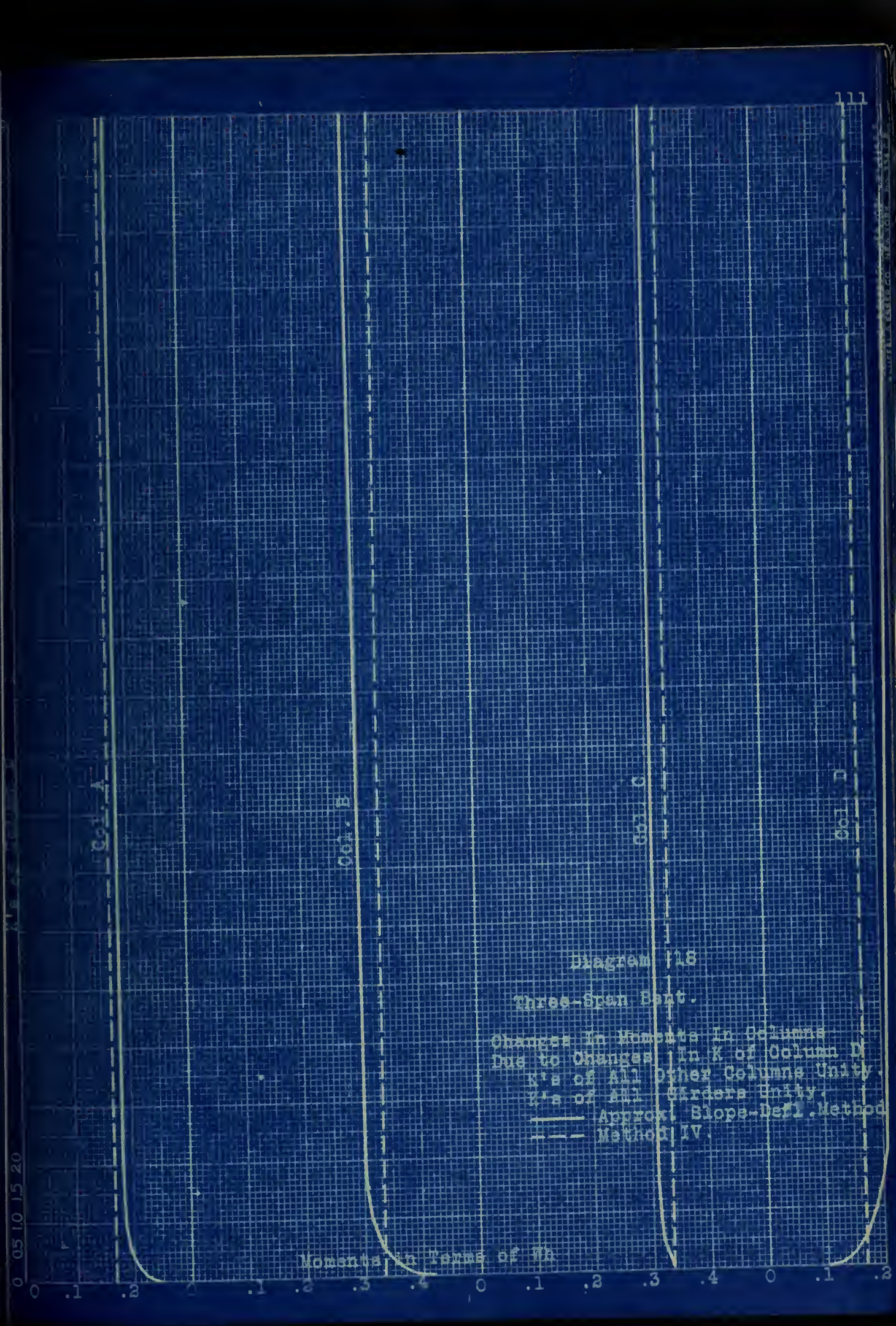
Two-Span Bent.
Moments In Column A (Diagram 15)
And
Moments In Column B (Diagram 16)

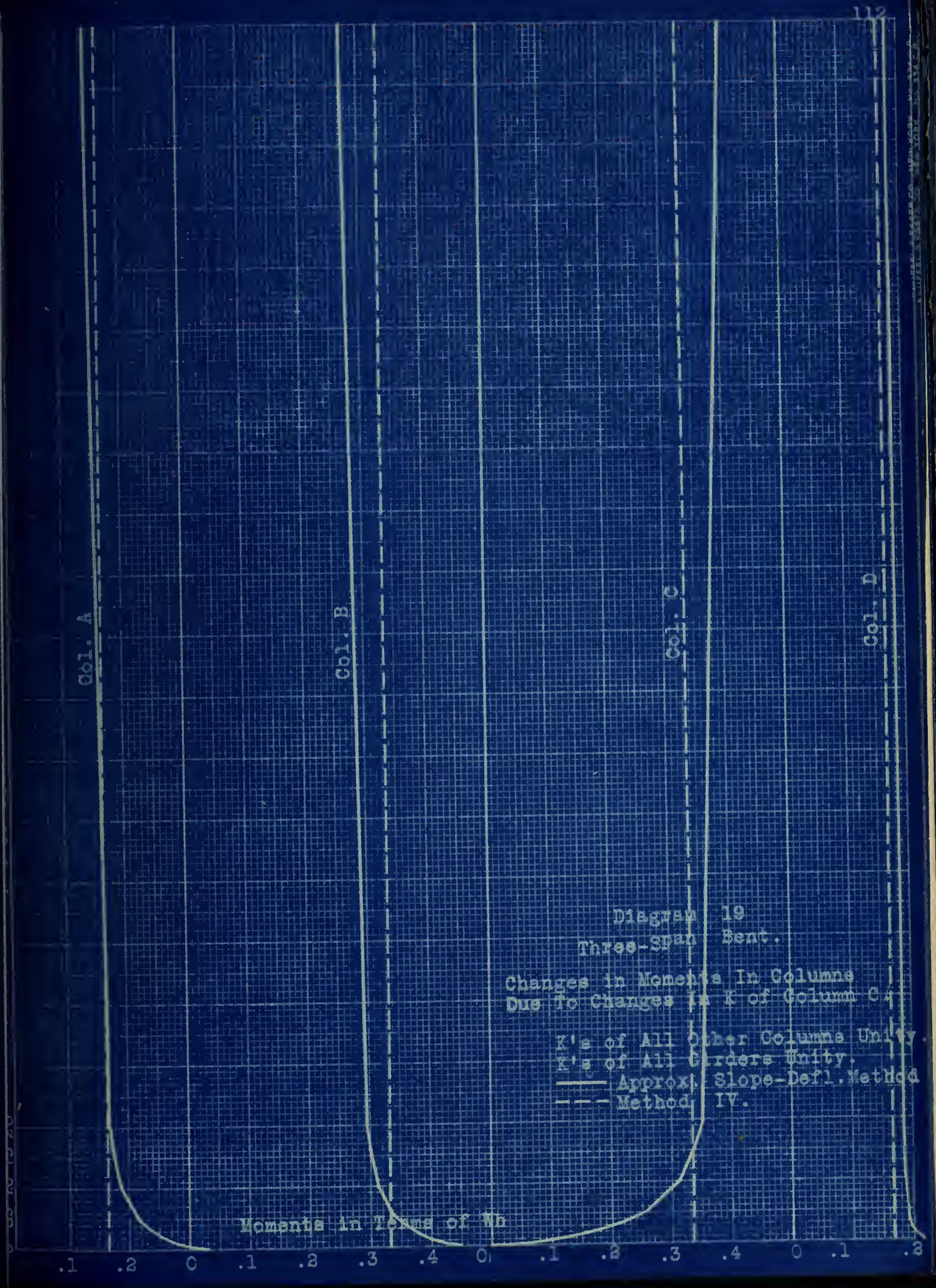
K's of Column C and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves Indicate magnitude of K for Members which vary.

THREE-SPAN BENT.

Diagrams 17 to 33, inclusive.


Col. A
Col. B
Col. C
Col. D
Diagram IV.
Three-Span Bent.
Changes in Moments in Columns
Due To Changes in K's Of Girders.
K's of All Columns Unity.
K's of All Girders Equal.
Approx. Slope-Defl. Method
Method IV.
Moments in Terms of Wh²
.1 .2 0 .1 .2 .3 .4 0 .1 .2 .3 .4 0 .1 .2


Col. A
Col. B
Col. C
Col. D
Diagram 118
Three-Span Bent.
Changes in Moments in Columns due to Changes in K of Column D
K's of All Other Columns Unity.
K's of All Girders Unity.
Approxi Slope-Defl. Method
Method IV.
Moments in Terms of Wh
0 .1 .2
0 .1 .2 .3 .4
0 .1 .2 .3 .4
0 .1 .2


Col. A
Col. B
Col. C
Col. D
Diagram 19
Three-Span Bent.
Changes in Moments In Columns
Due To Changes In K of Column C4
K's of All Other Columns Unity.
K's of All Girders Unity.
Approx. Slope-Defl. Method
Method IV.
Moments in Terms of Wh
.1 .2 0 .1 .2 .3 .4 0 .1 .2 .3 .4 0 .1 .2

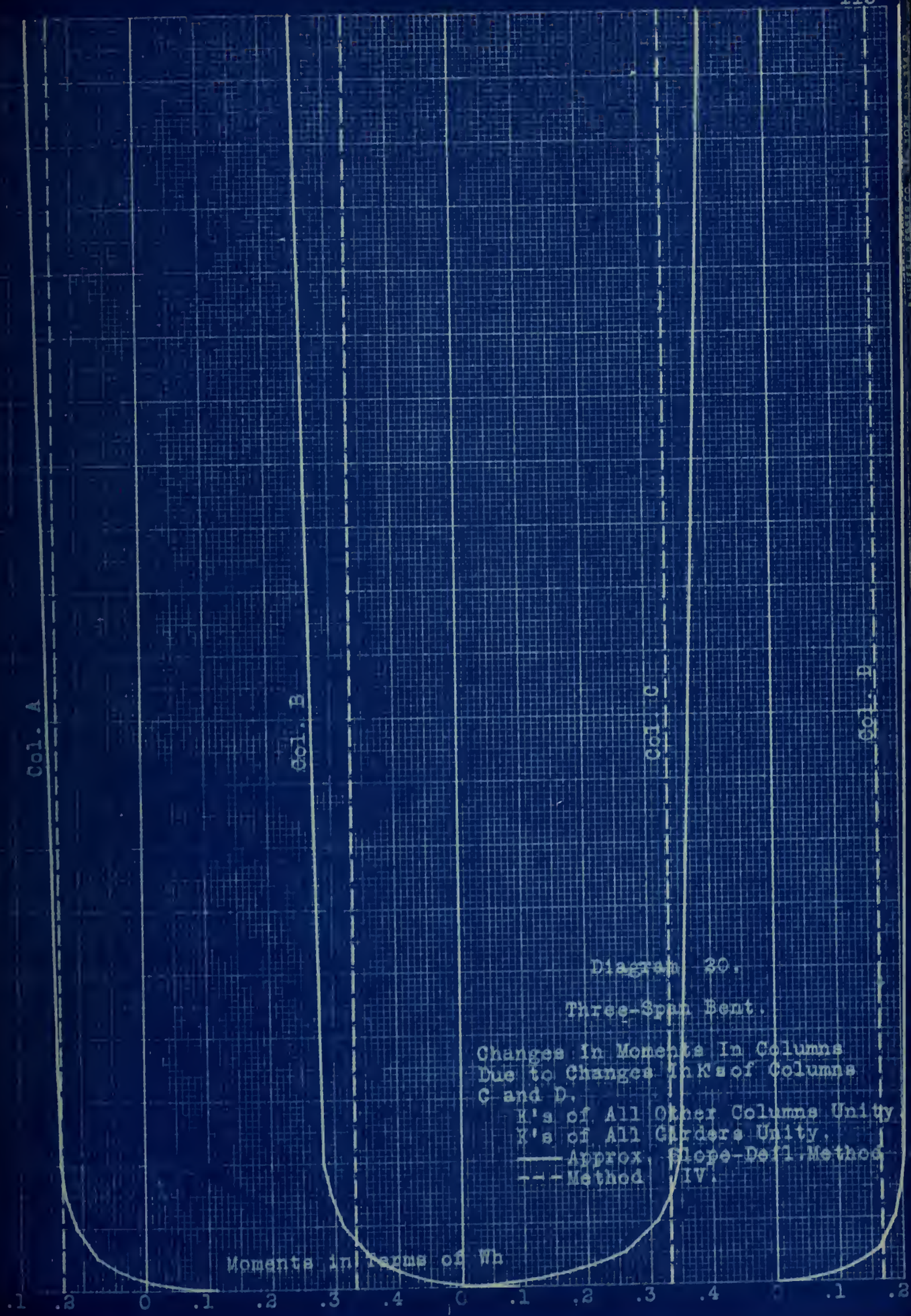
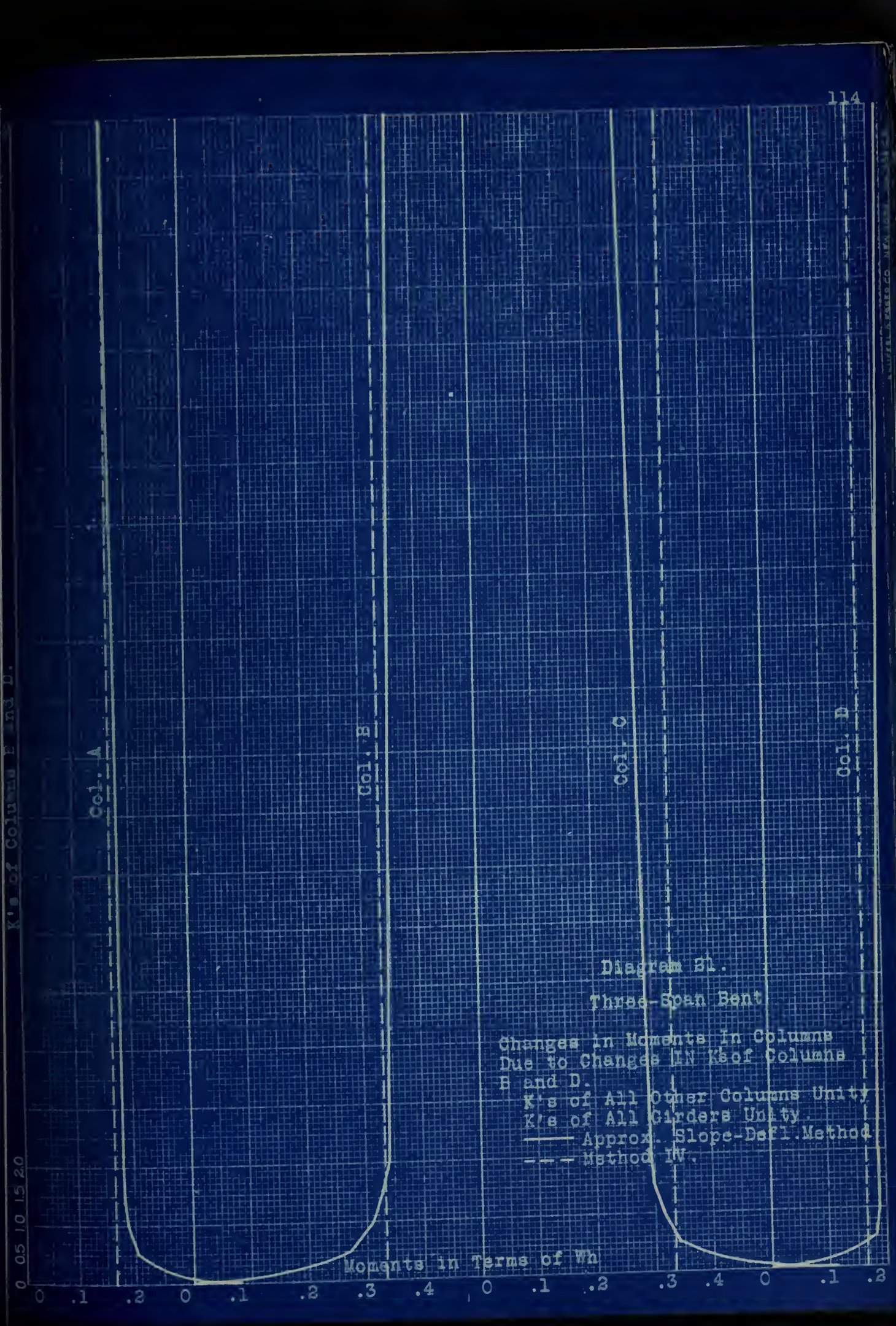


Diagram 30.

Three-Span Bent.

Changes In Moments In Columns Due to Changes In K's of Columns C and D.

K's of All Other Columns Unity.
K's of All Girders Unity.
——— Approx. Slope-Defl. Method
- - - Method IV.


Col. A
Col. B
Col. C
Col. D
Diagram B1.
Three-Span Bent
Changes in Moments in Columns
Due to Changes IN K's of Columns
B and D.
K's of All Other Columns Unity
K's of All Girders Unity.
Approx. Slope-Defl. Method
Method IV.
Moments in Terms of Wh
0 .1 .2 0 .1 .2 .3 .4 0 .1 .2 .3 .4 0 .1 .2
0 .05 .10 .15 .20
K's of Columns B and D.

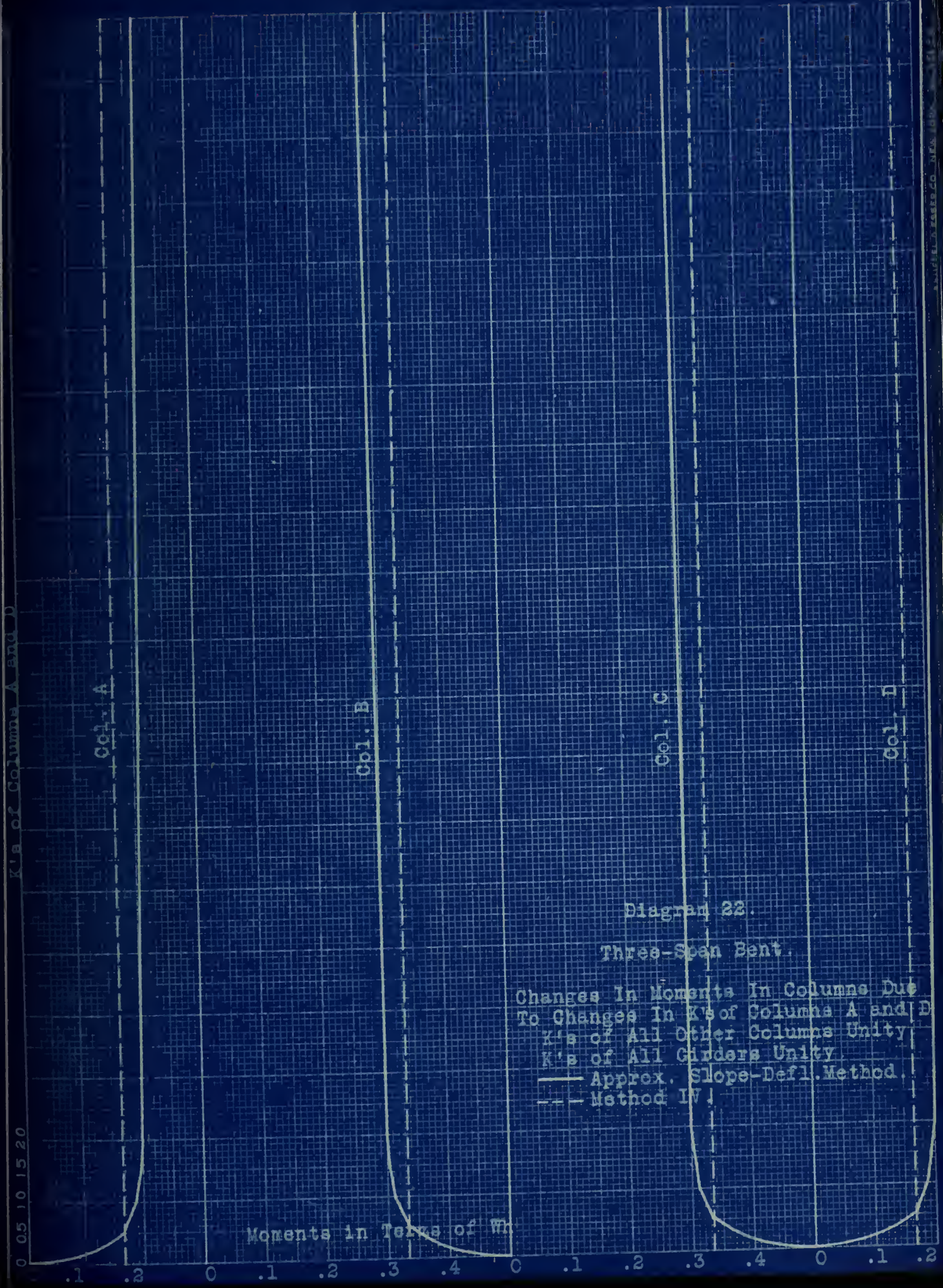


Diagram 22.

Three-Span Bent.

Changes In Moments In Columns Due To Changes In K's of Columns A and D
K's of All Other Columns Unity
K's of All Girders Unity.
—— Approx. Slope-Defl. Method
- - - Method IV

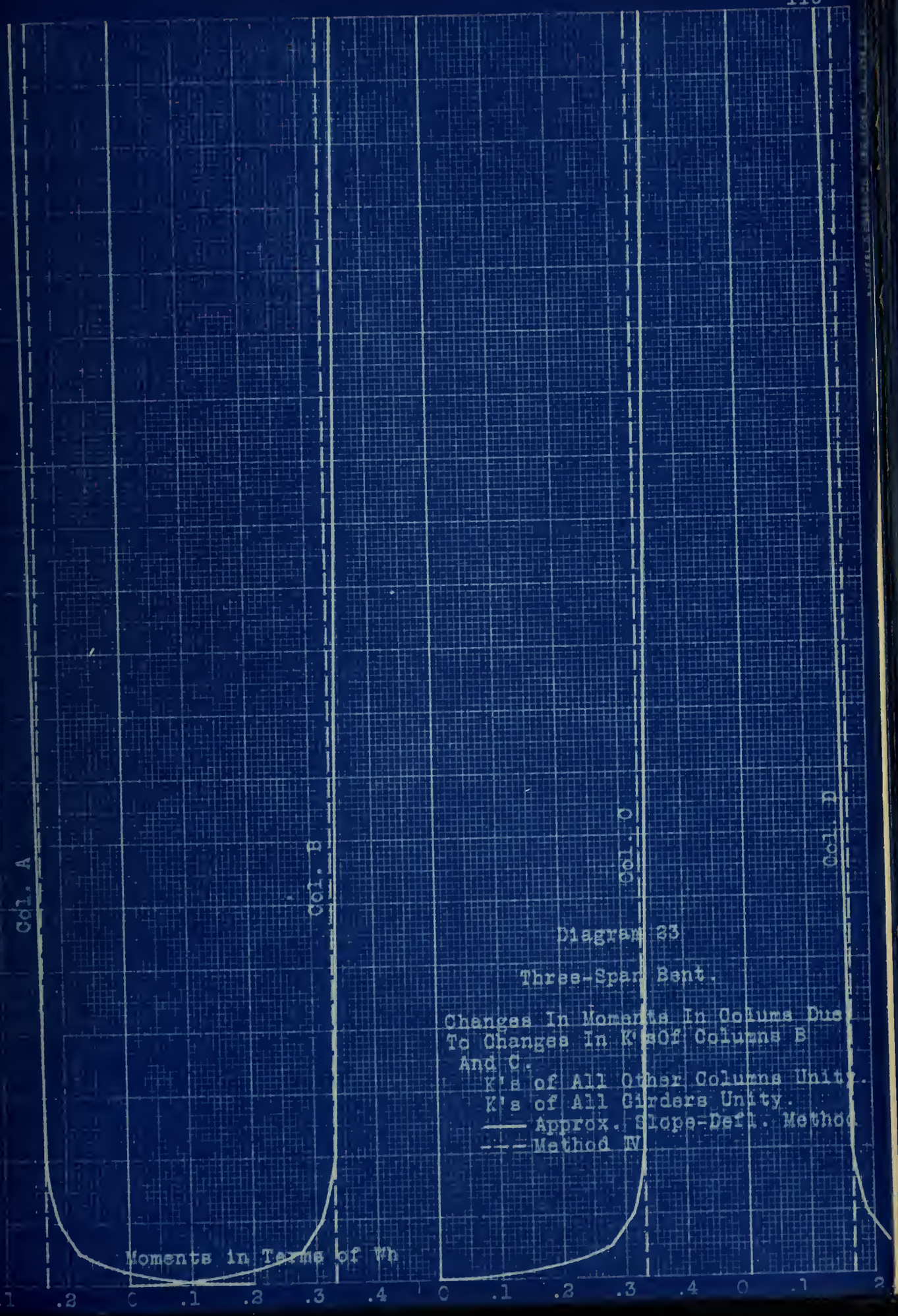


Diagram 23

Three-Span Bent.

Changes In Moments In Colums Due To Changes In K's Of Columns B And C.

K's of All Other Columns Unity.

K's of All Girders Unity.

—— Approx. Slope-Defl. Method

- - - Method IV

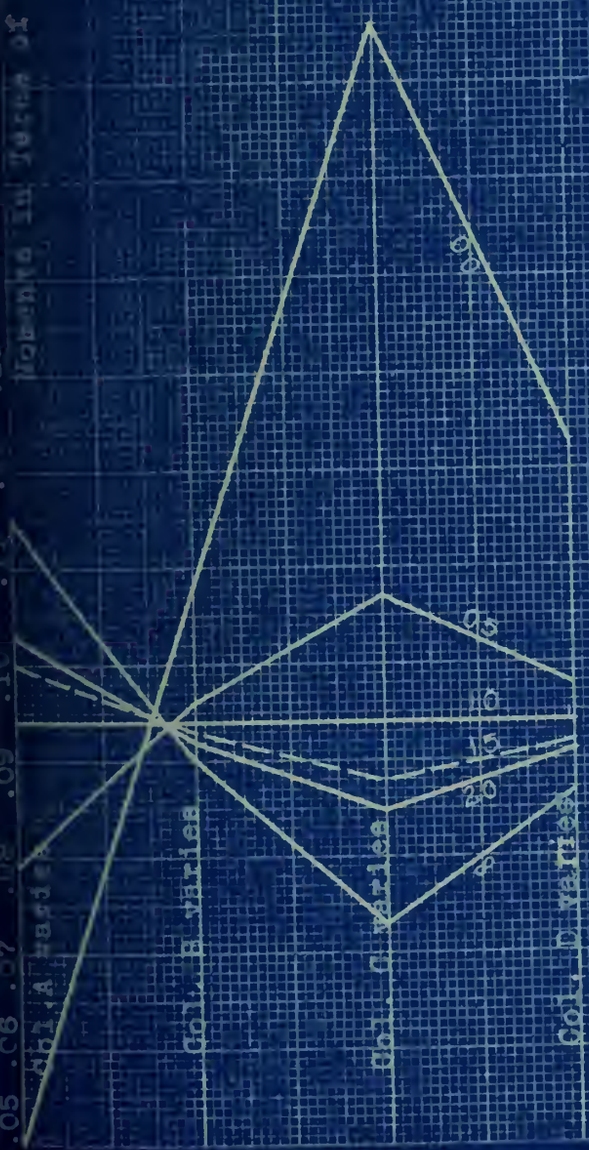


Diagram 24

Three-Span Bent.

Moments in Column A.
K of One Column Changes.
K's of Other Columns Unity.
K of Girder Unity.
Numbers on Curves indicate magnitudes of K for member which varies.

Three-Span Bent.

Moments in Column A.

Diagram 25; K's of Column A and One Other Column Change Simultaneously.
Diagram 28; K's of Column B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K for members which vary.

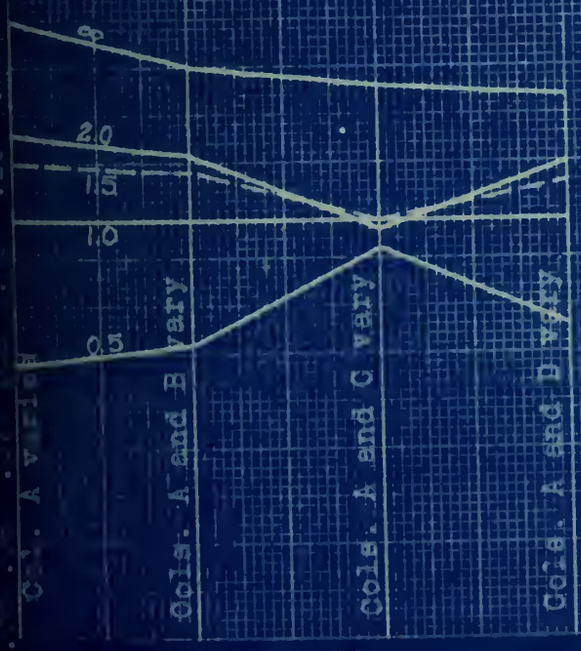


Diagram 25

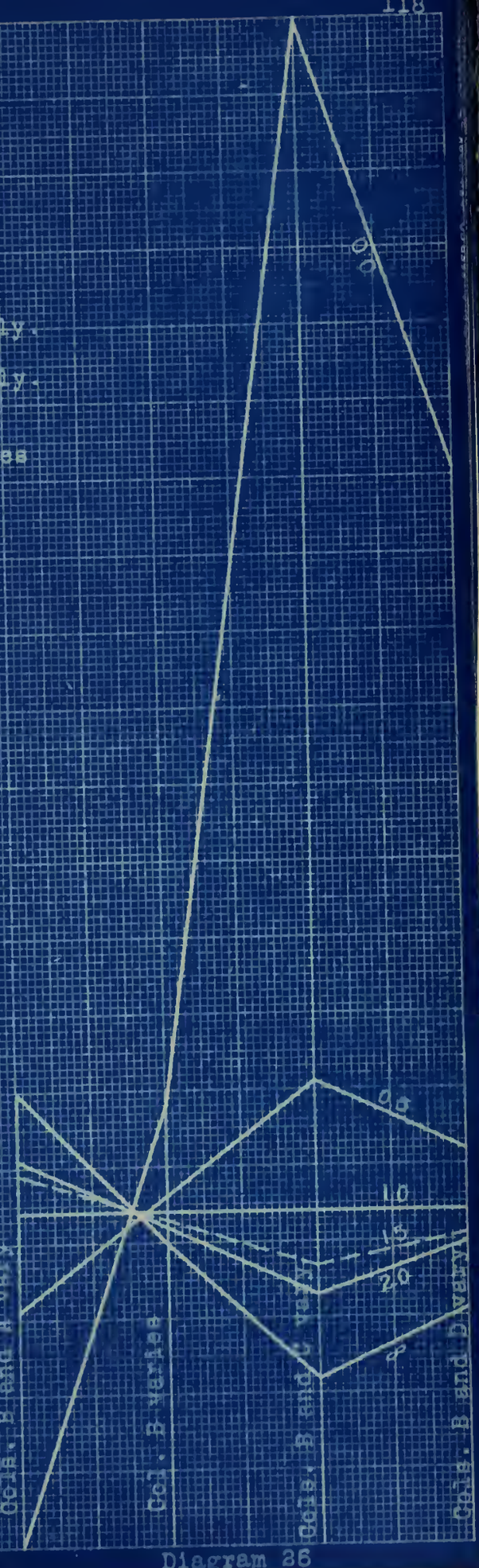


Diagram 26

Three Span Bent.

Moments In Column A.

Diagram 27: K's of Column C and One Other Column Change Simultaneously.
Diagram 28: K's of Column D and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K for members which vary.

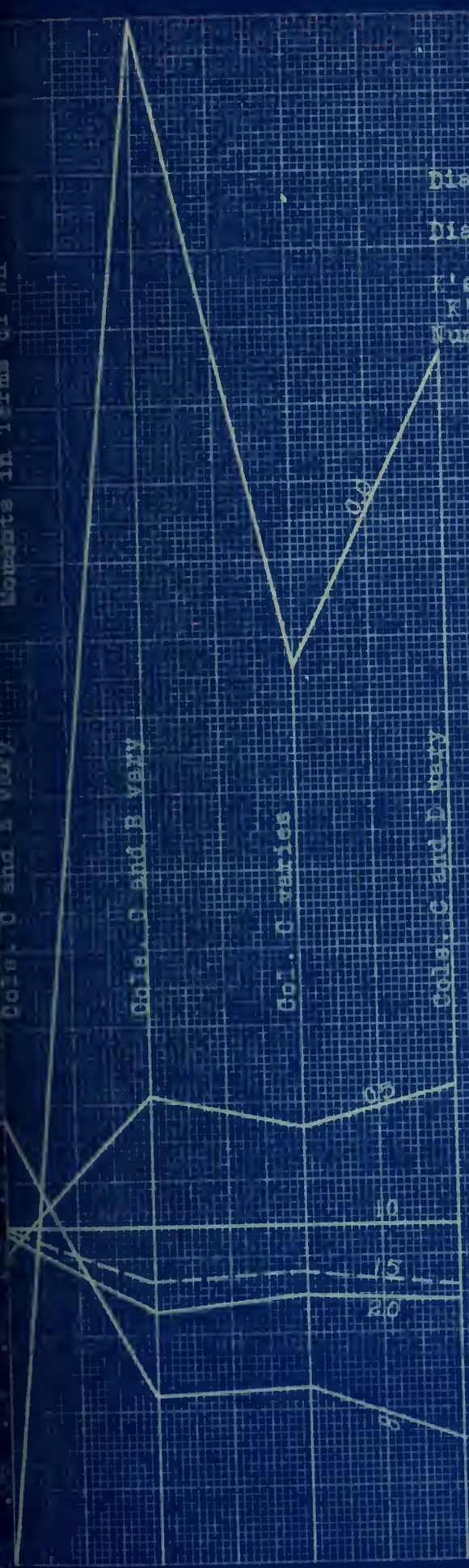


Diagram 37

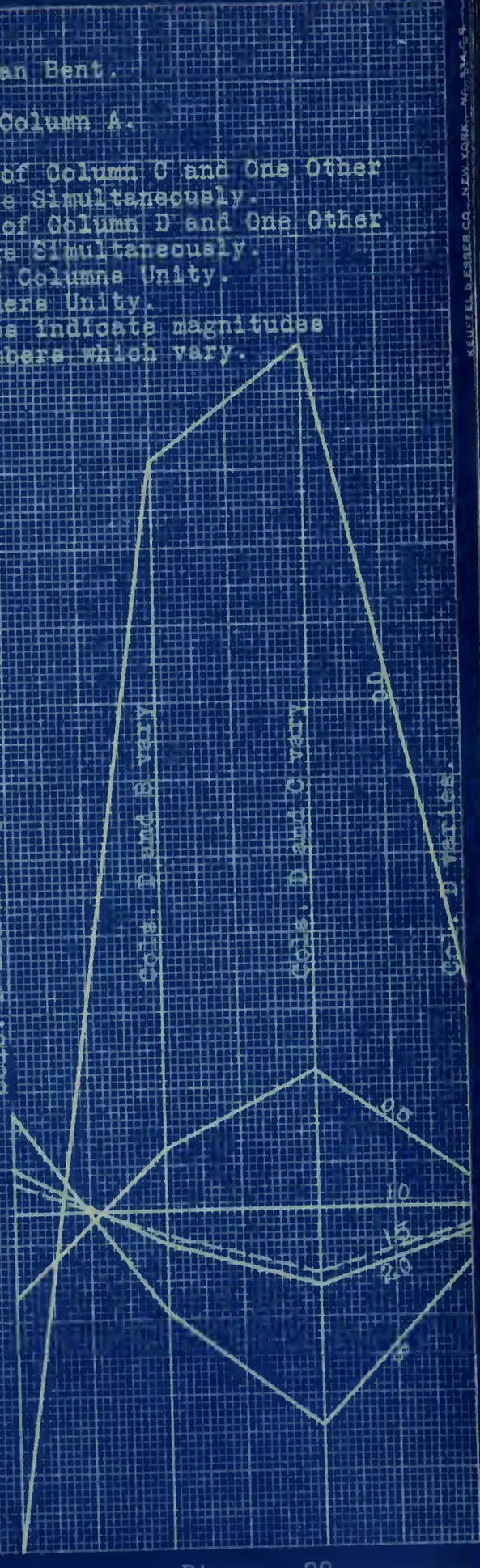


Diagram 28

Diagram 29.

Three-Span Bent.

Moments in Column B.
K of One Column Changes.
K's of Other Columns Unity.
K of Girder Unity.
Numbers on Curves indicate magnitudes of K for member which varies.

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Three-Span Bent.

Moments in Column B.

Diagram 30: K's of Column A and One Other Column Change Simultaneously.
Diagram 31: K's of Column B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K for members which vary.

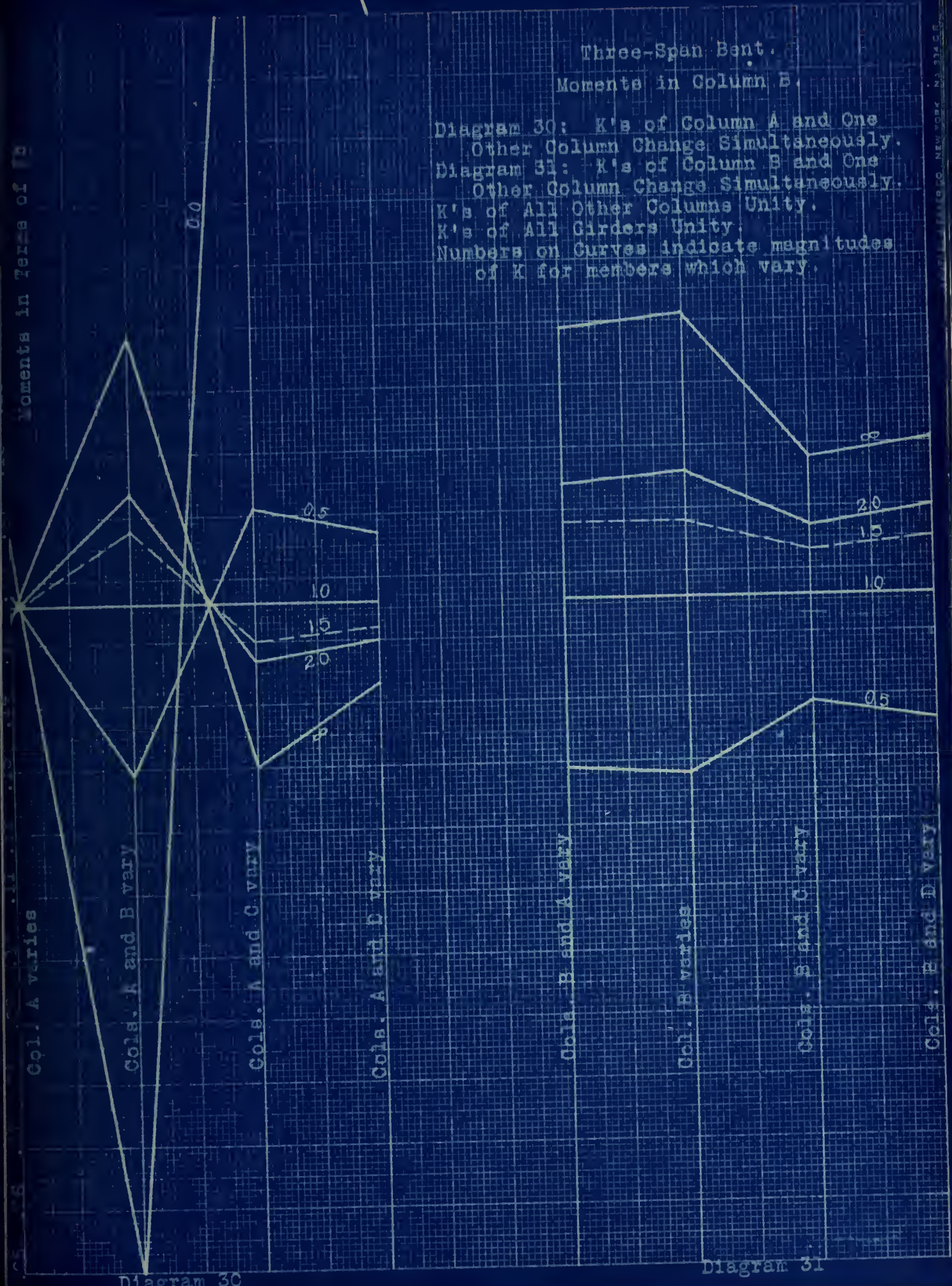
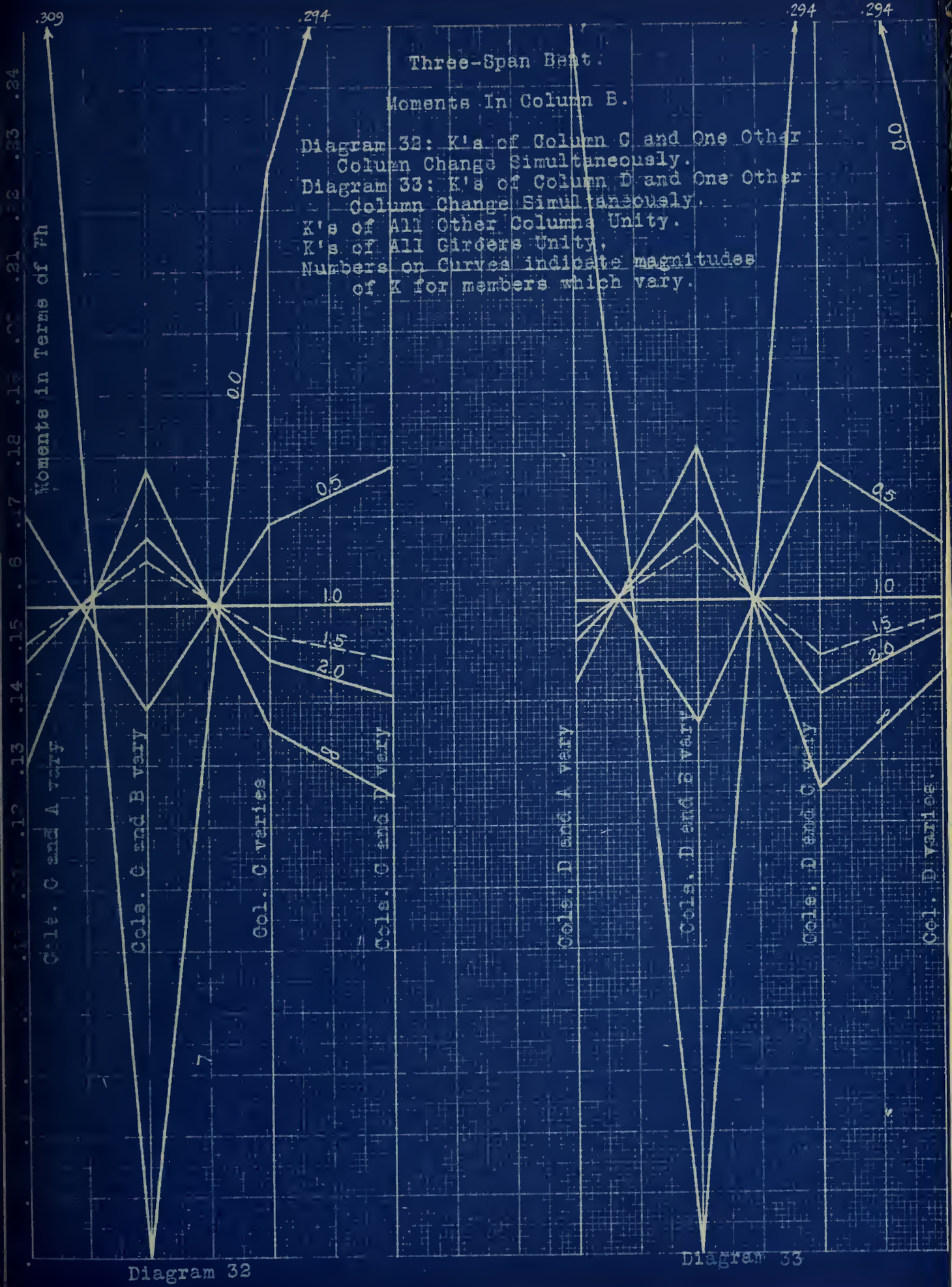


Diagram 30

Diagram 31


Three-Span Bent.
Moments In Column B.
Diagram 32: K's of Column C and One Other Column Change Simultaneously.
Diagram 33: K's of Column D and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K for members which vary.
Moments in Terms of Fh
.309
.294
.294
.294
.24
.23
.22
.21
.18
.17
.15
.14
.13
.12
.6
0.0
0.5
1.0
1.5
2.0
∞
Cols. C and A vary
Cols. C and B vary
Col. C varies
Cols. C and D vary
Cols. D and A vary
Cols. D and B vary
Cols. D and C vary
Col. D varies
Diagram 32
Diagram 33

FOUR-SPAN BENT.

Diagrams 34 to 61, inclusive.

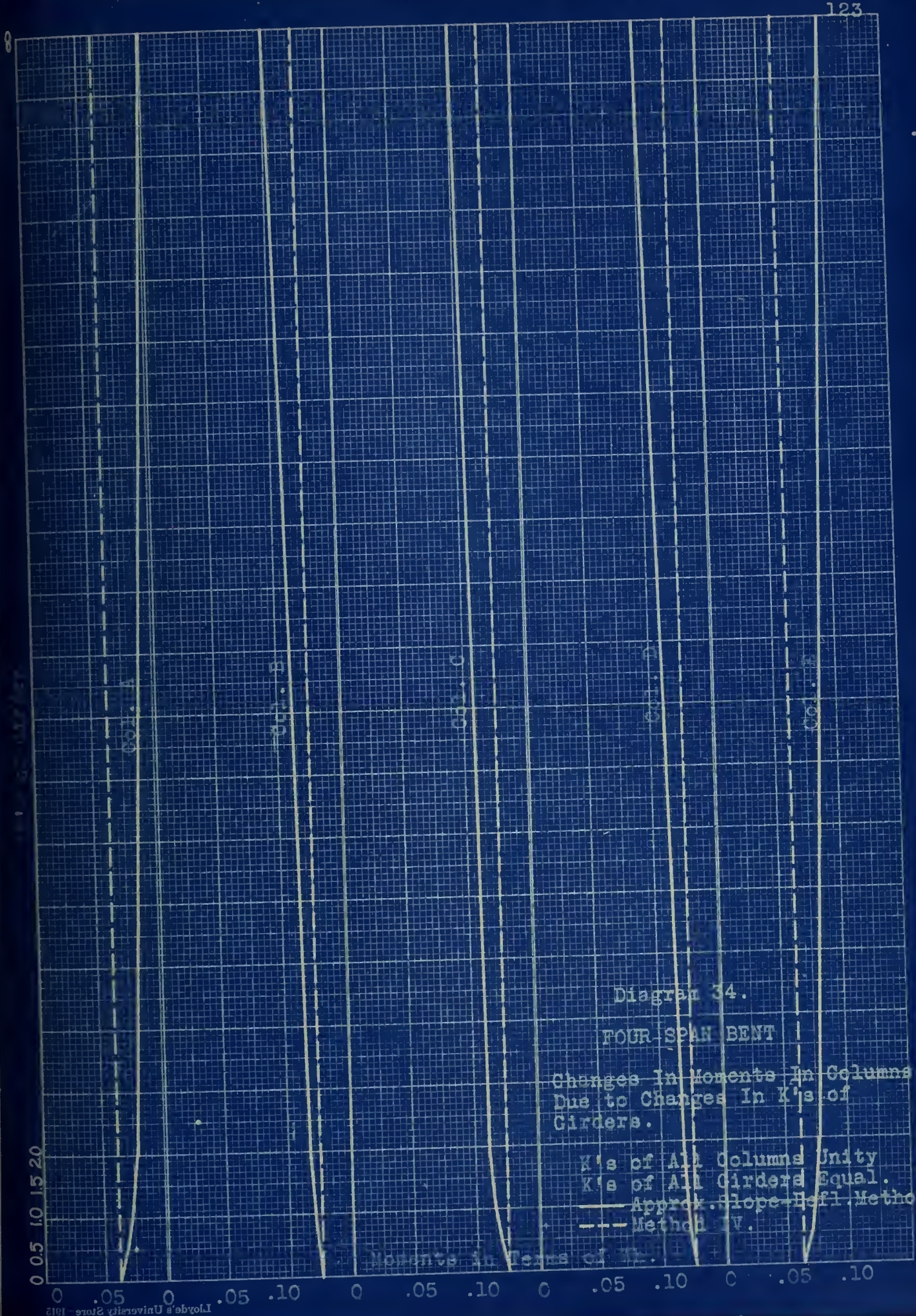
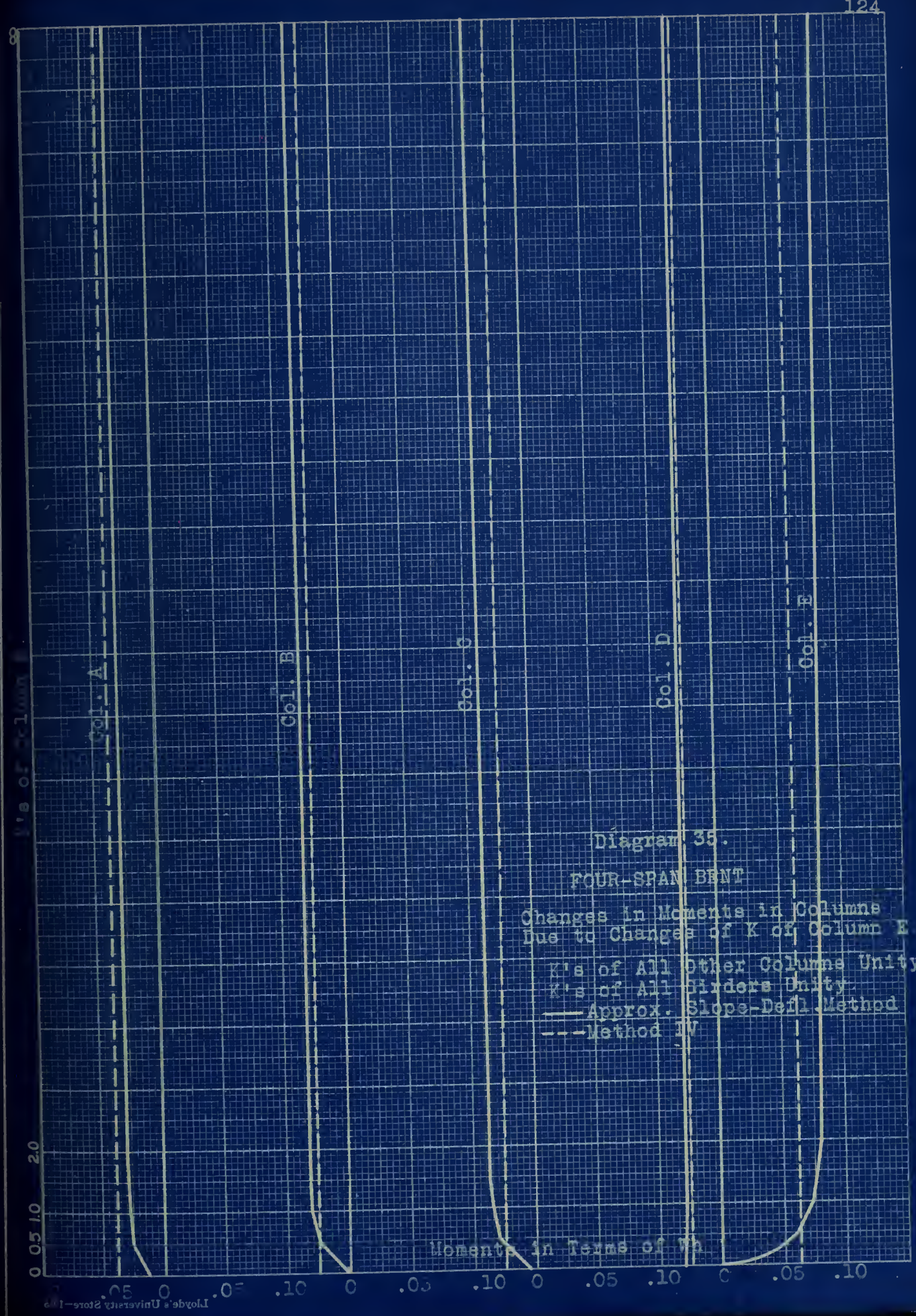
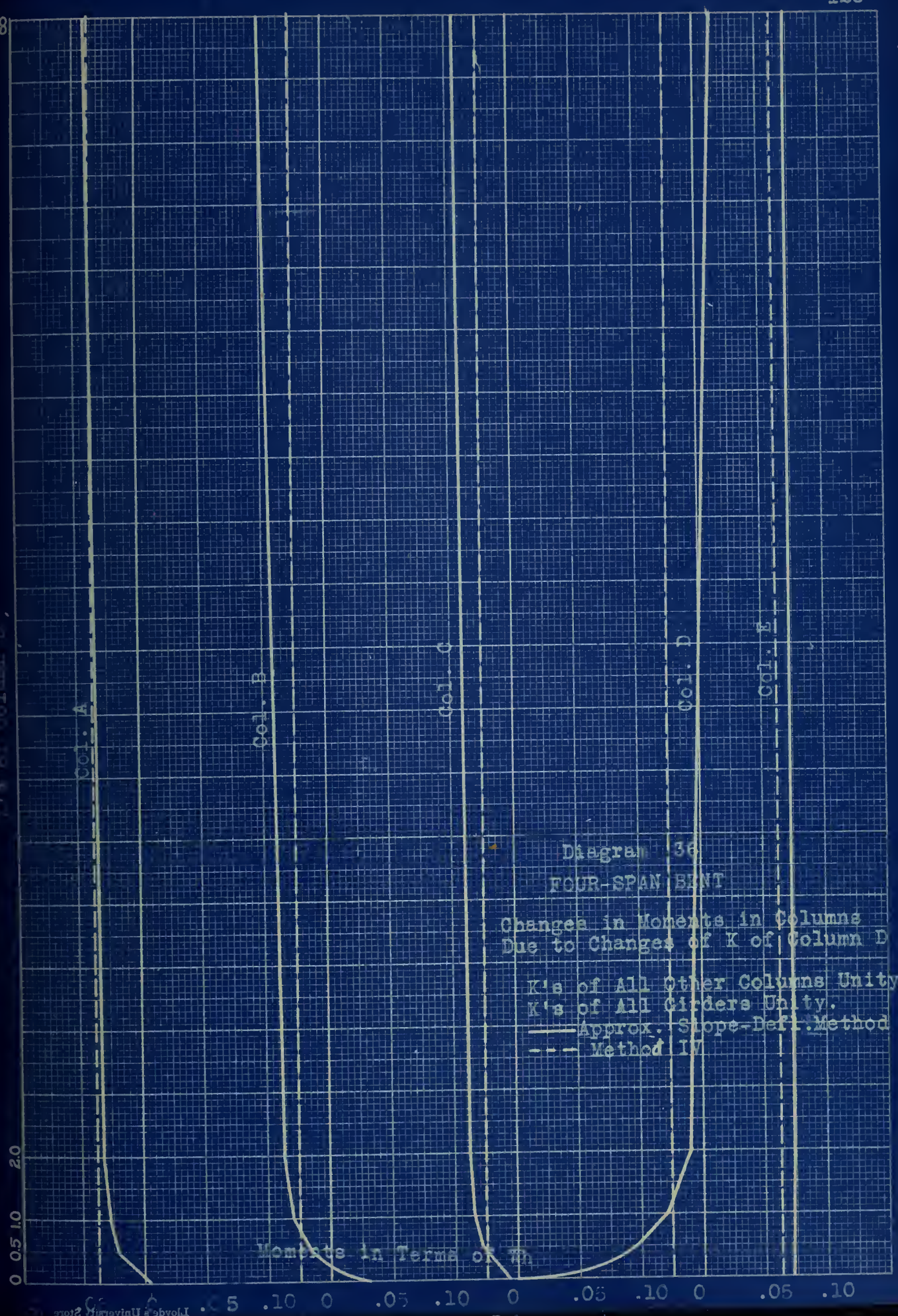
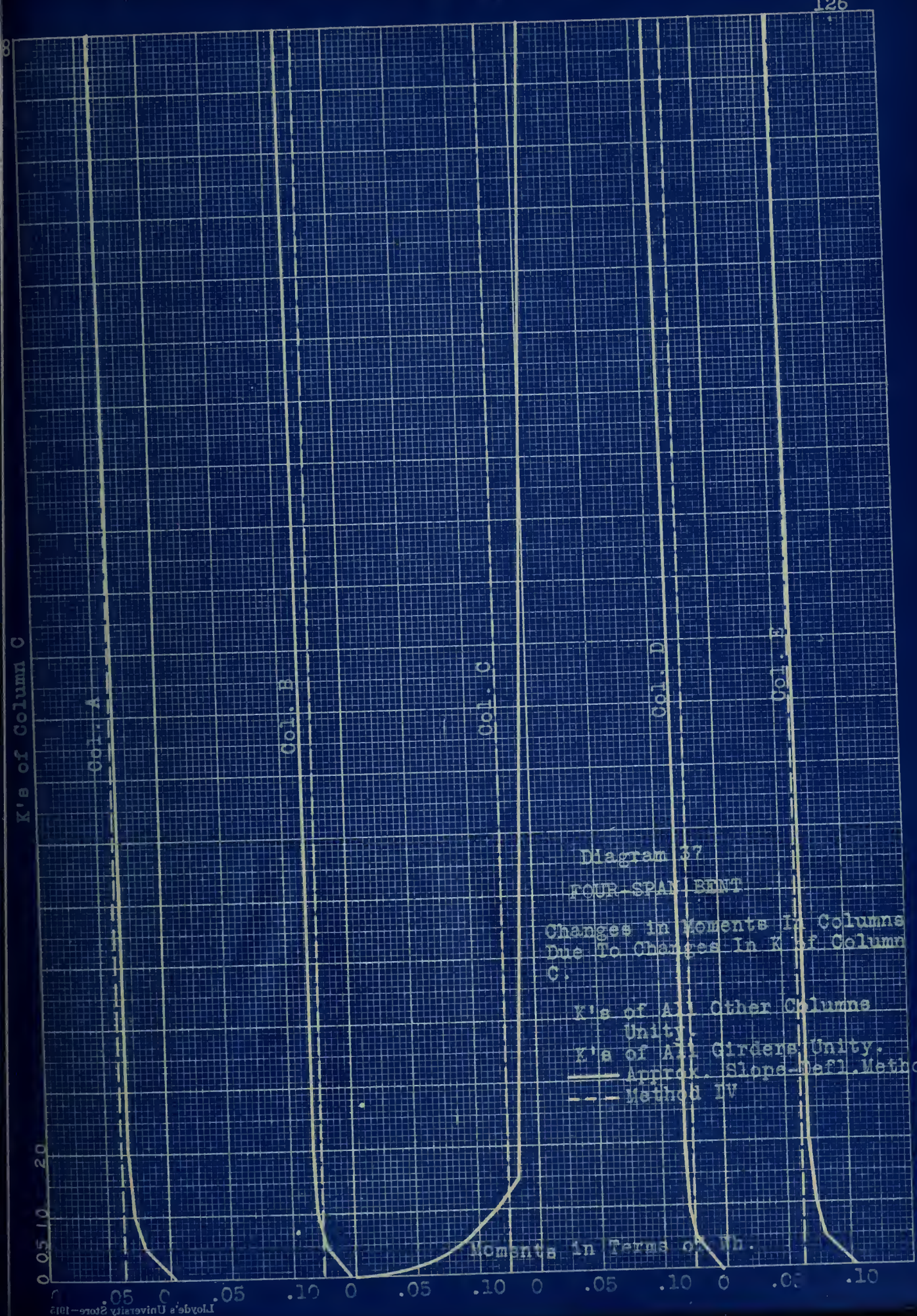


Diagram 34.
FOUR-SPAN BENT
Changes In Moments In Columns Due to Changes In K's of Girders.
K's of All Columns Unity
K's of All Girders Equal.
Approx. Slope-Defl. Method
Method IV.
Moments in Terms of M.
0 .05 0 .05 .10 0 .05 .10 0 .05 .10 0 .05 .10
0 0.5 1.0 1.5 2.0


K's of Column E
Col. A
Col. B
Col. C
Col. D
Col. E
Diagram 35.
FOUR-SPAN BENT
Changes in Moments in Columns
Due to Changes of K of Column E
K's of All Other Columns Unity.
K's of All Girders Unity.
——Approx. Slope-Defl. Method
---Method IV
Moments in Terms of Wh
0 .05 1.0 2.0
.05 0 .05 .10 0 .05 .10 0 .05 .10 0 .05 .10


Col. A
Col. B
Col. C
Col. D
Col. E
Diagram 36
FOUR-SPAN BENT
Changes in Moments in Columns
Due to Changes of K of Column D
K's of All Other Columns Unity
K's of All Girders Unity.
——Approx. Slope-Defl. Method
- - - Method IV
Moments in Terms of m
.05 .10 0 .05 .10 0 .05 .10 0 .05 .10



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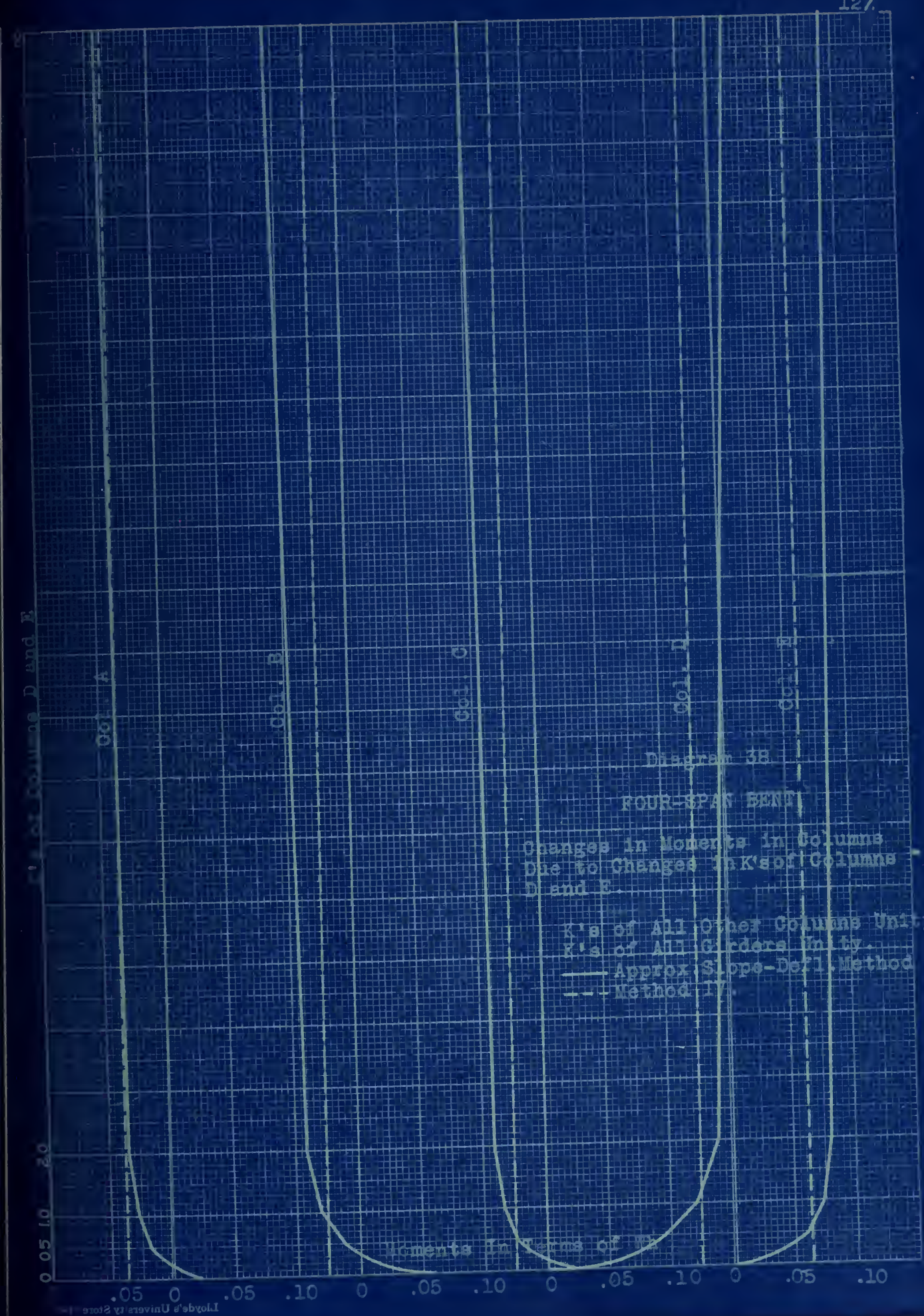
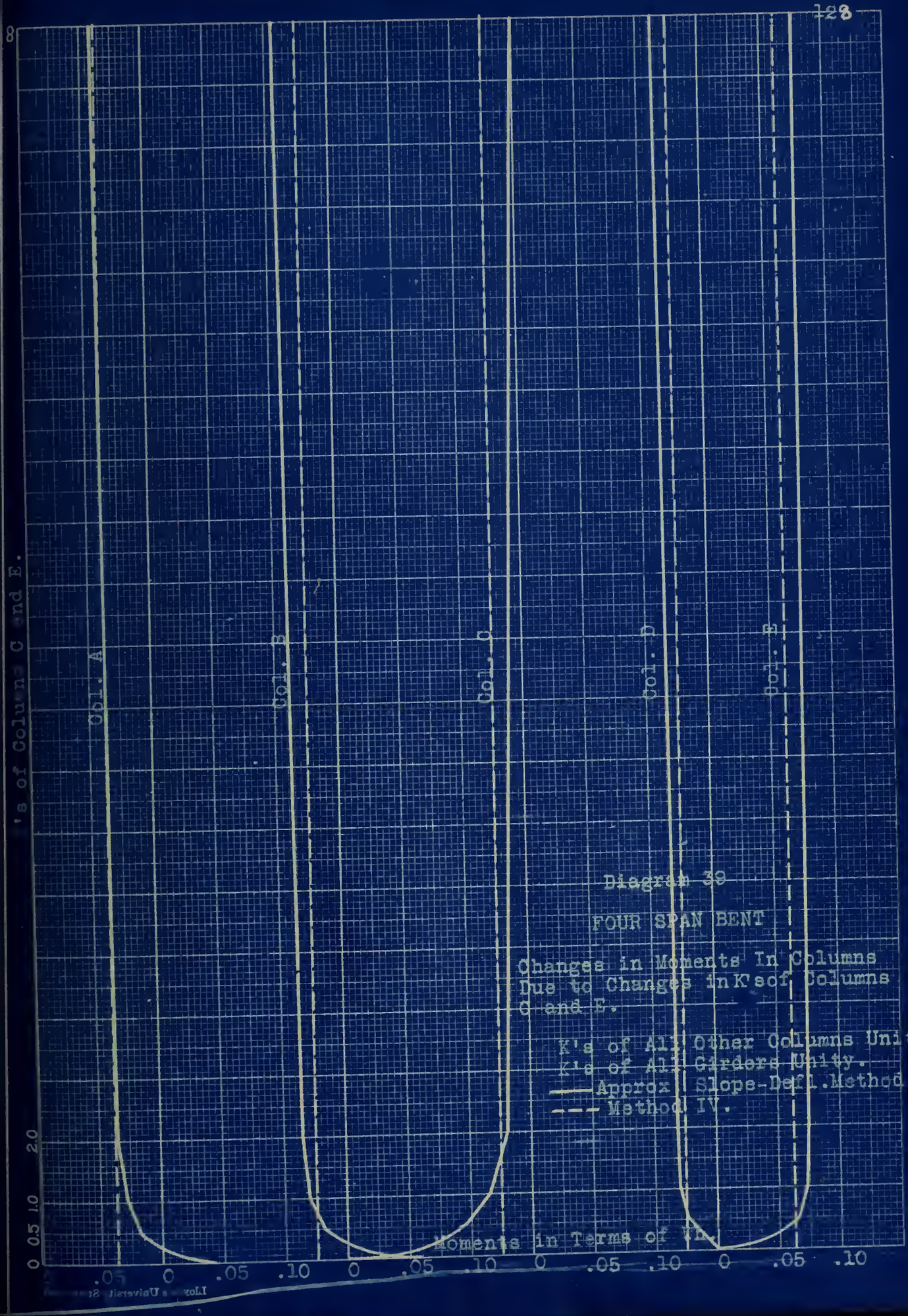

Col. A
Col. B
Col. C
Col. D
Col. E
Diagram 38
FOUR-SPAN BENT
Changes in Moments in Columns Due to Changes in K's of Columns D and E
K's of All Other Columns Unity
K's of All Girders Unity.
Approx. Slope-Defl. Method
Method IV
Moments in Terms of wL²
.05 0 .05 .10 0 .05 .10 0 .05 .10
0 .05 .10 .20


Col. A
Col. B
Col. C
Col. D
Col. E
Diagram 39
FOUR SPAN BENT
Changes in Moments In Columns Due to Changes in K's of Columns C and E.
K's of All Other Columns Unity
K's of All Girders Unity.
Approx. Slope-Defl. Method
Method IV.
Moments in Terms of
.05 0 .05 .10 0 .05 .10 0 .05 .10 0 .05 .10
0 0.5 1.0 2.0

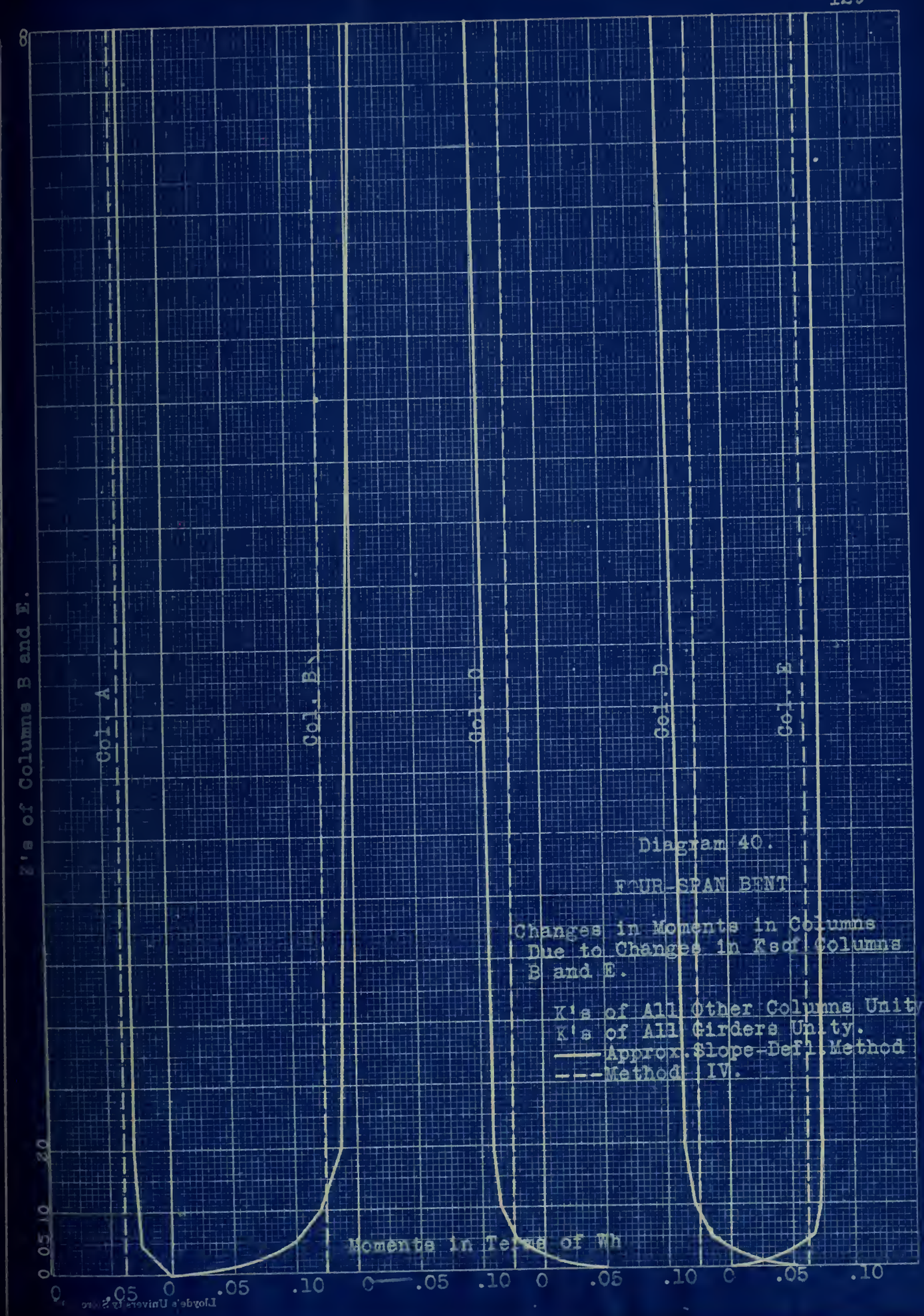


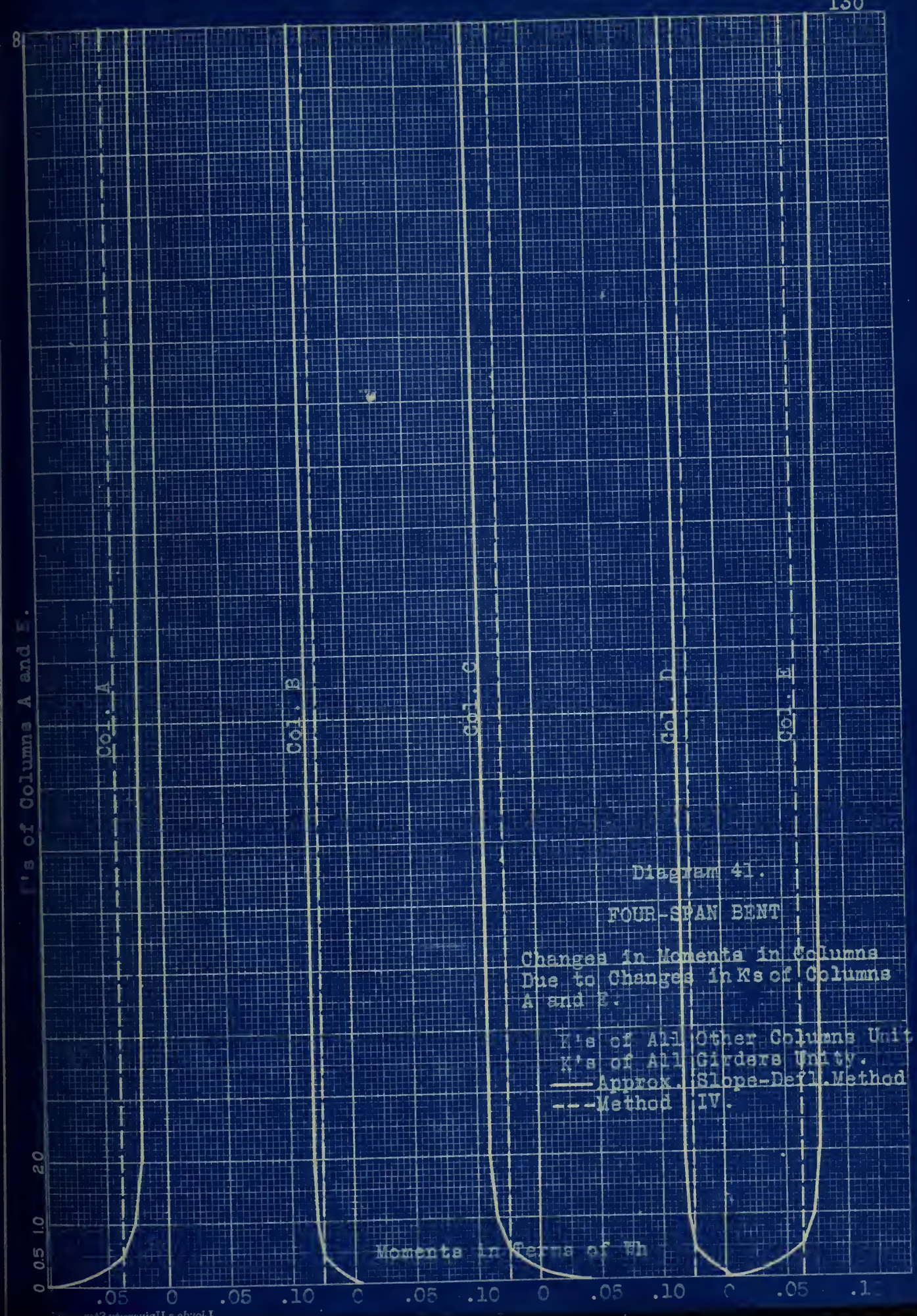
Diagram 40.

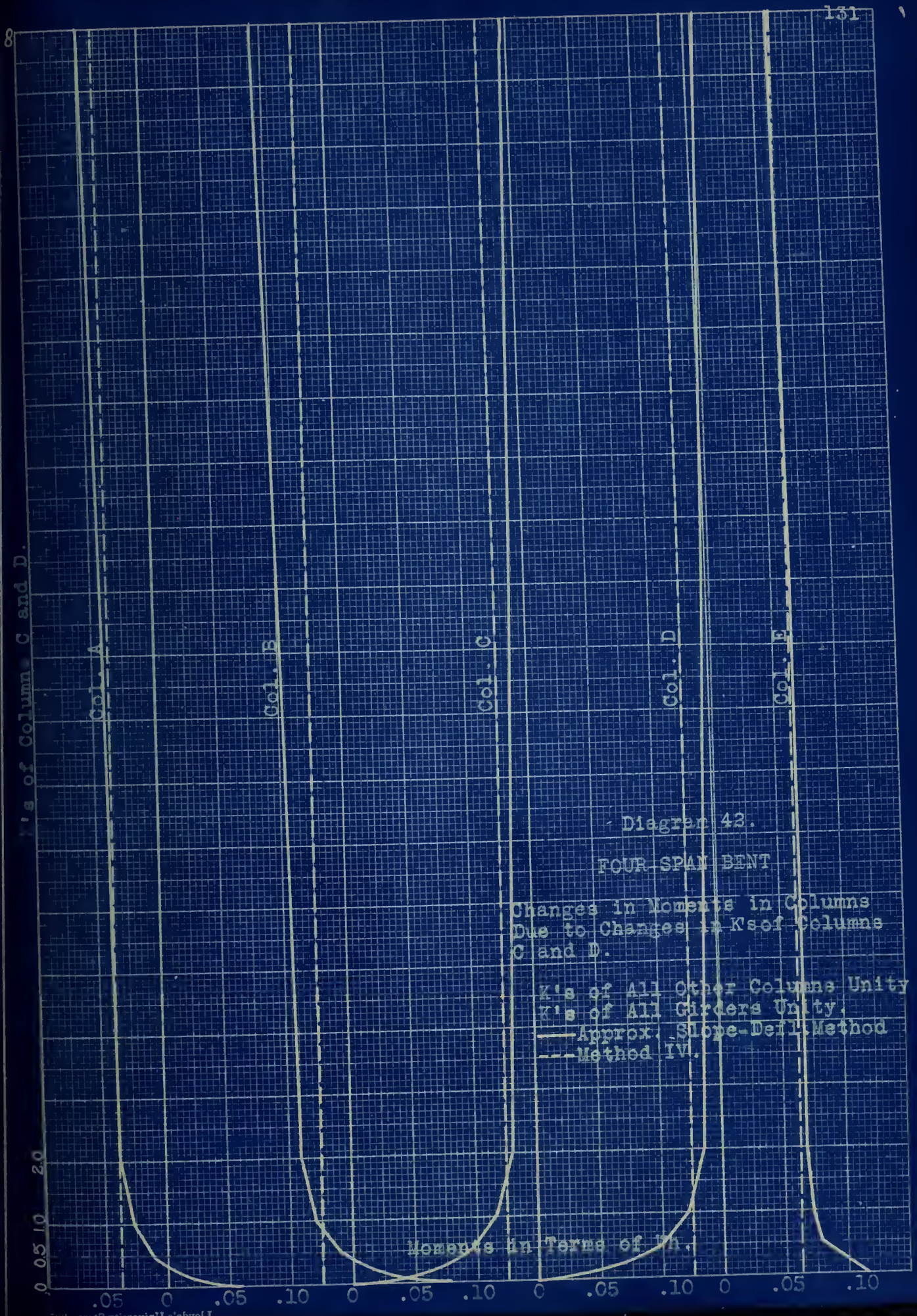
FOUR-SPAN BENT

Changes in Moments in Columns Due to Changes in K's of Columns B and E.

K's of All Other Columns Unity
K's of All Girders Unity.
——Approx. Slope-Defl. Method
- - -Method IV.

Lloyde's University Library


Col. A
Col. B
Col. C
Col. D
Col. E
Moments in Terms of Wh
K's of Columns A and E.
0 0.5 1.0 2.0
.05 0 .05 .10 0 .05 .10 0 .05 .10 0 .05 .1
Diagram 41.
FOUR-SPAN BENT
Changes in Moments in Columns Due to Changes in Ks of Columns A and E.
K's of All Other Columns Unity
K's of All Girders Unity.
——Approx. Slope-Defl. Method
- - -Method IV.


Col. A
Col. B
Col. C
Col. D
Col. E
Diagram 43.
FOUR SPAN BENT
Changes in Moments in Columns Due to Changes in K's of Columns C and D.
K's of All Other Columns Unity
K's of All Girders Unity.
——Approx. Slope-Def. Method
- - -Method IV.
Moments in Terms of Wh.
K's of Column C and D.
0 0.5 1.0 2.0
.05 0 .05 .10 0 .05 .10 0 .05 .10

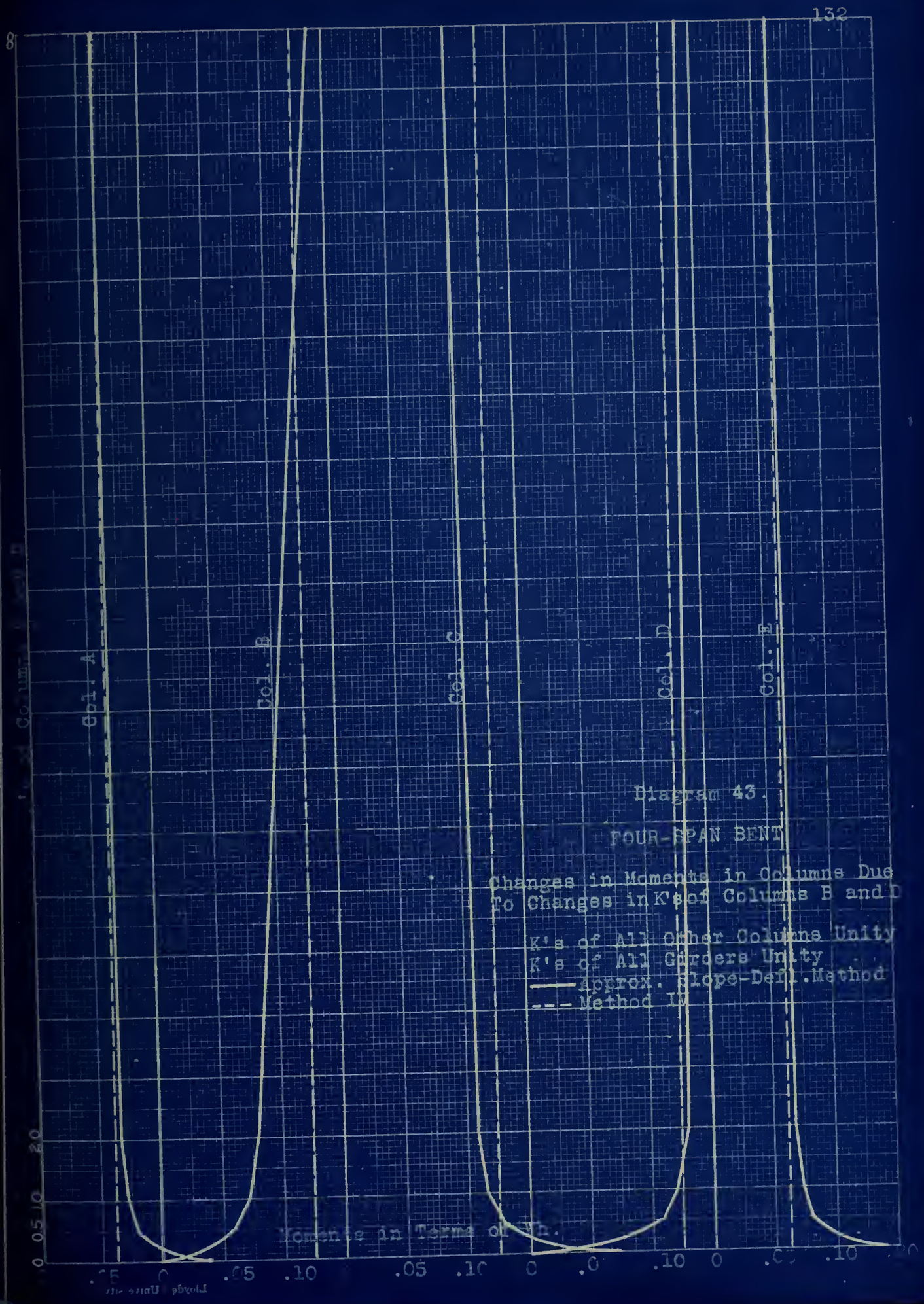


Diagram 43.

FOUR-SPAN BENT

Changes in Moments in Columns Due To Changes in K's of Columns B and D

K's of All Other Columns Unity
K's of All Girders Unity
——— Approx. Slope-Defl. Method
- - - Method II

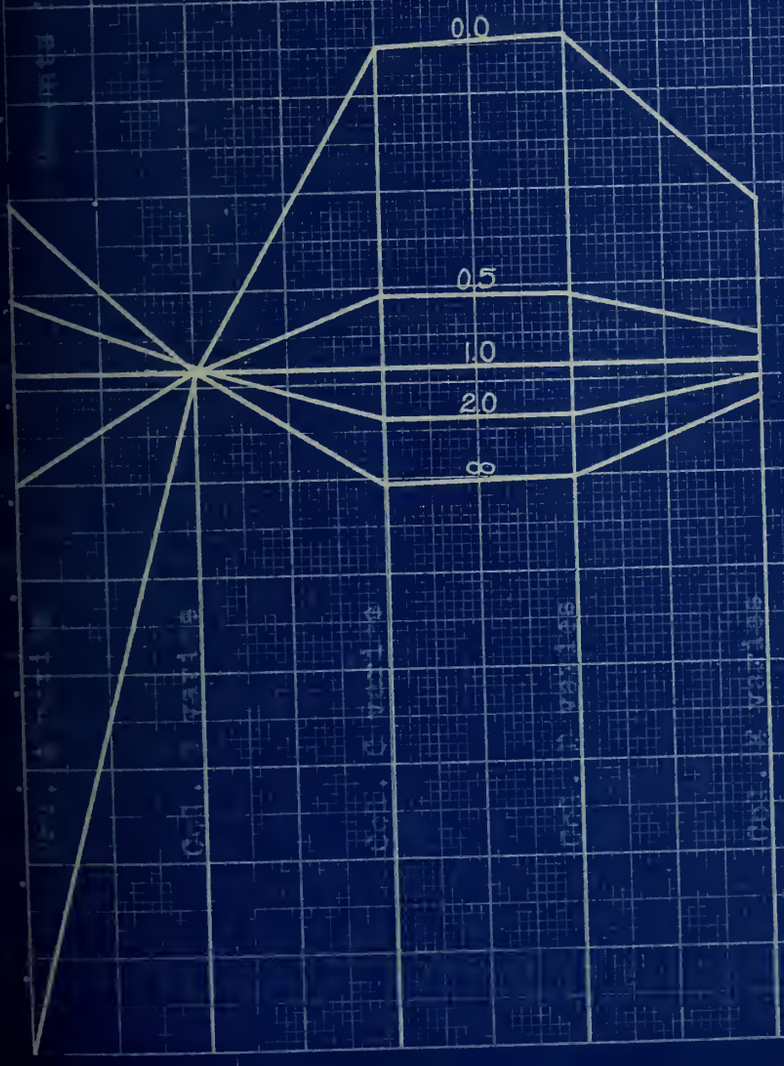


Diagram 44.

FOUR-SPAN BENT.

Moments in Column A.

K of One Column Changes
K's of Other Columns Unity
K of Girder Unity.
Numbers on Curves indicate
magnitudes of K for member
which varies.

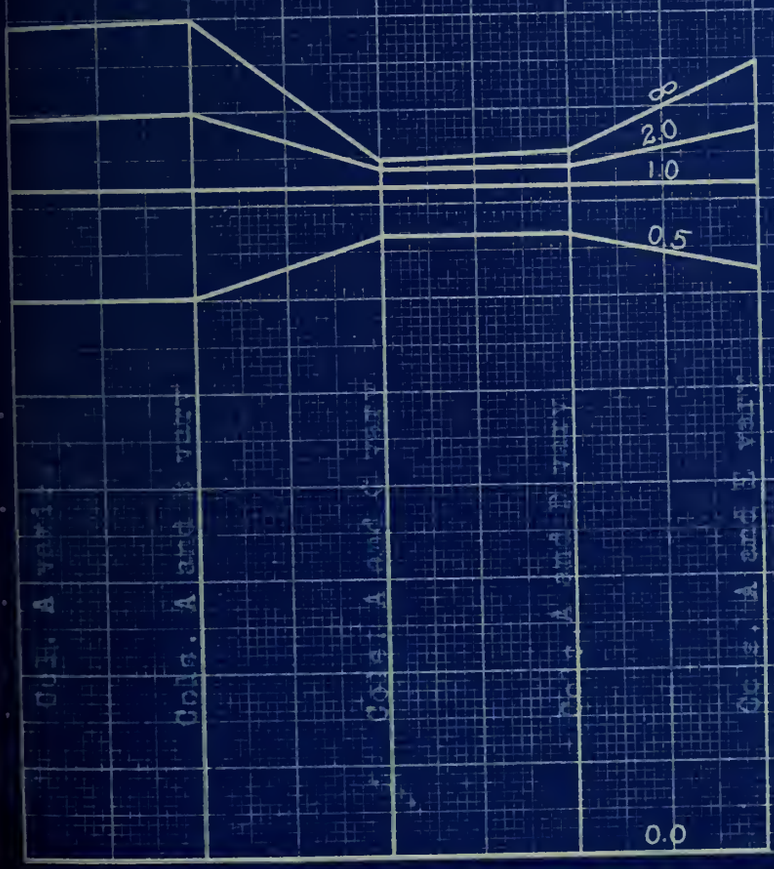


Diagram 45.

FOUR-SPAN BENT

Moments in Column A.

K's of Column A and One Other Column Change Simultaneously.
K's of All Other Columns Unity
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

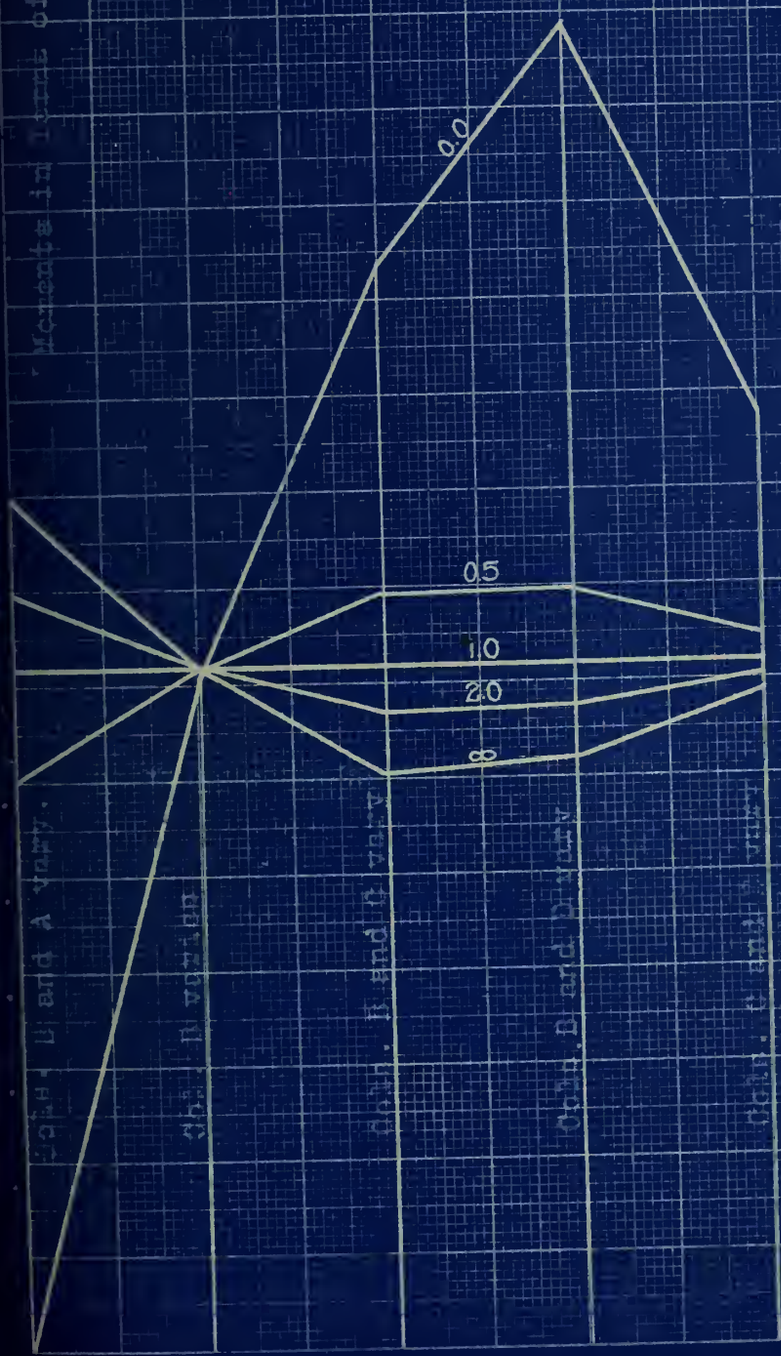


Diagram 46

FOUR-SPAN BENT.

Moments in Column A.

K's of Column B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

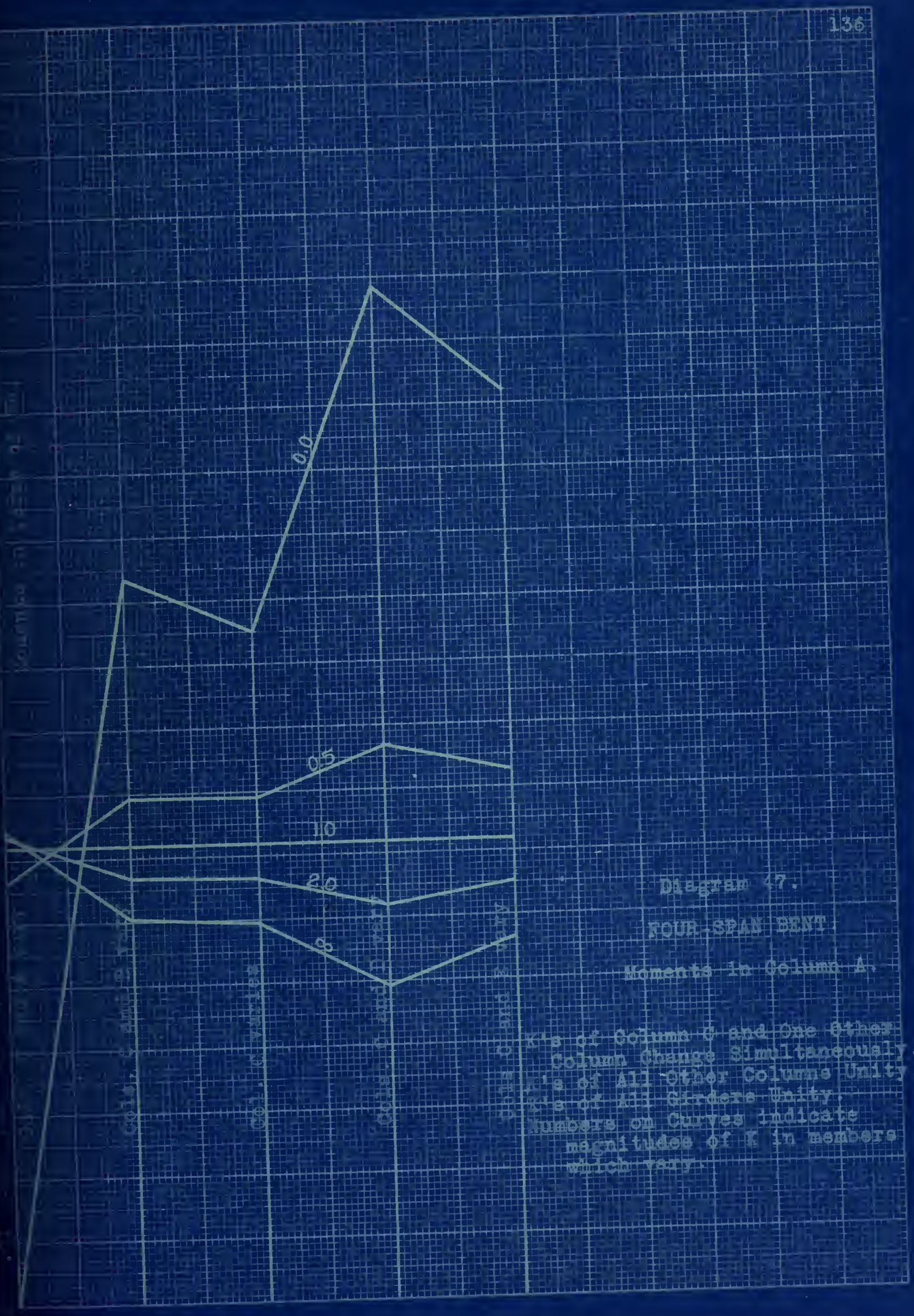


Diagram 17.

FOUR SPAN BENT.

Moments in Column A.

K's of Column C and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

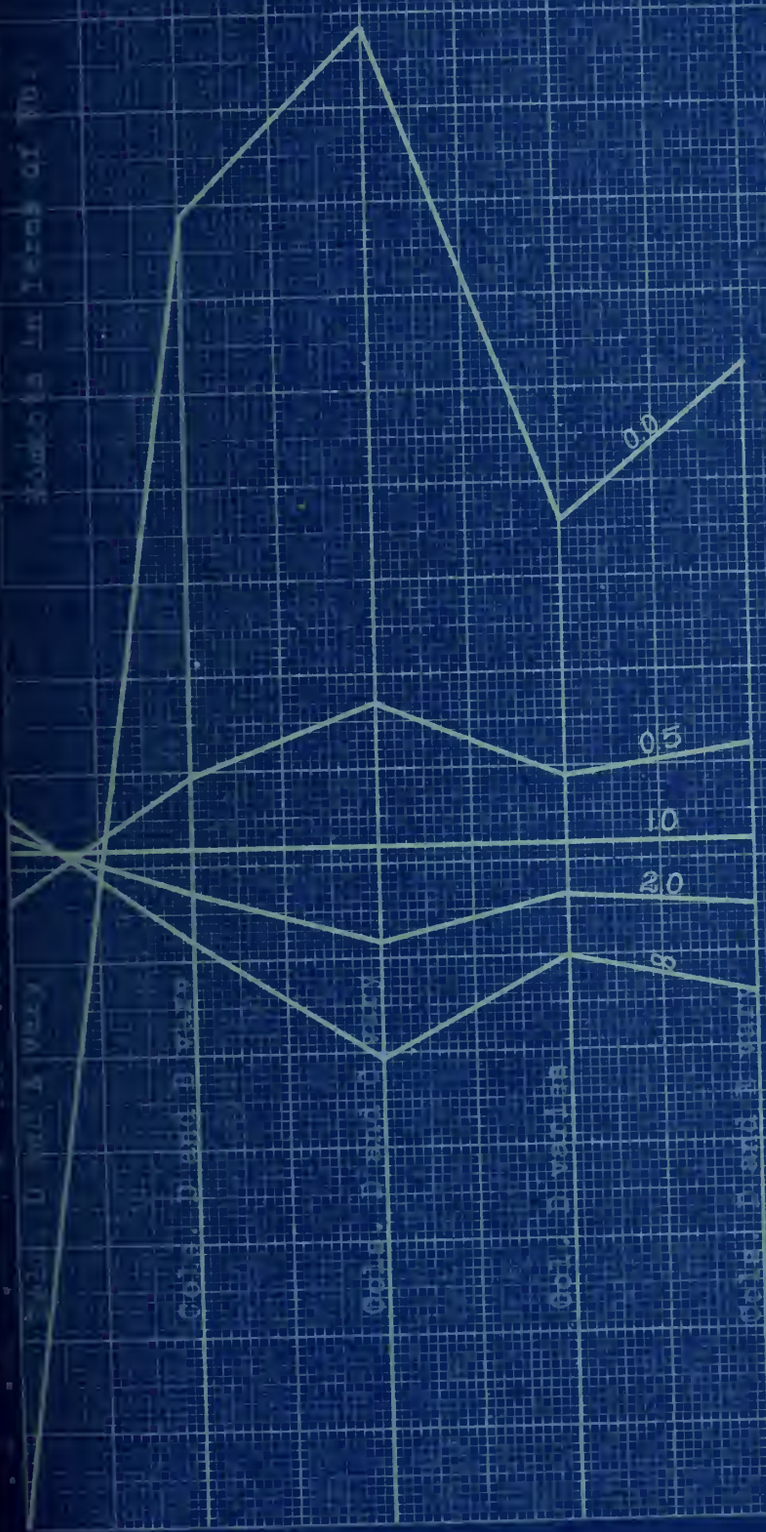


Diagram 46.

FOUR-SPAN BENT

Moments in Column A.

K's of Column B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves Indicate Magnitudes of K in members which vary.

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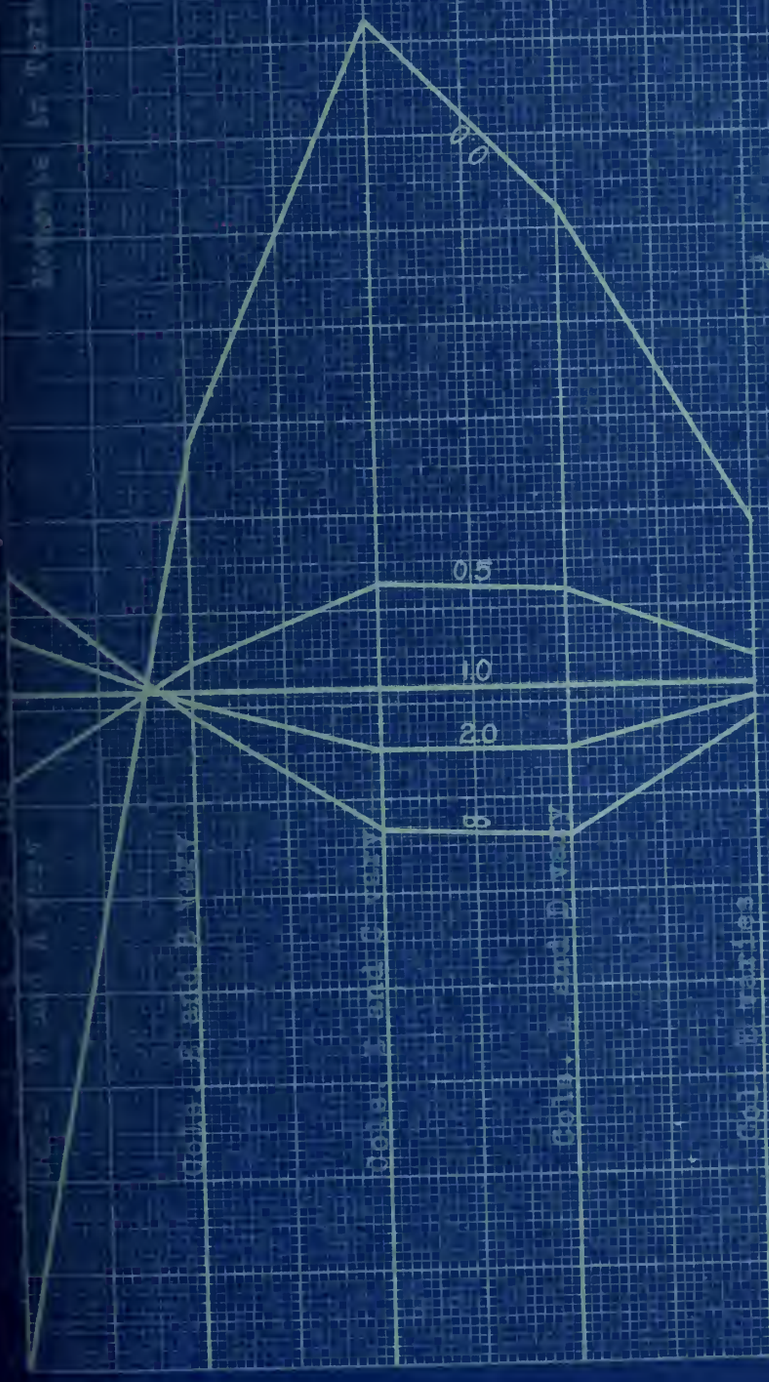

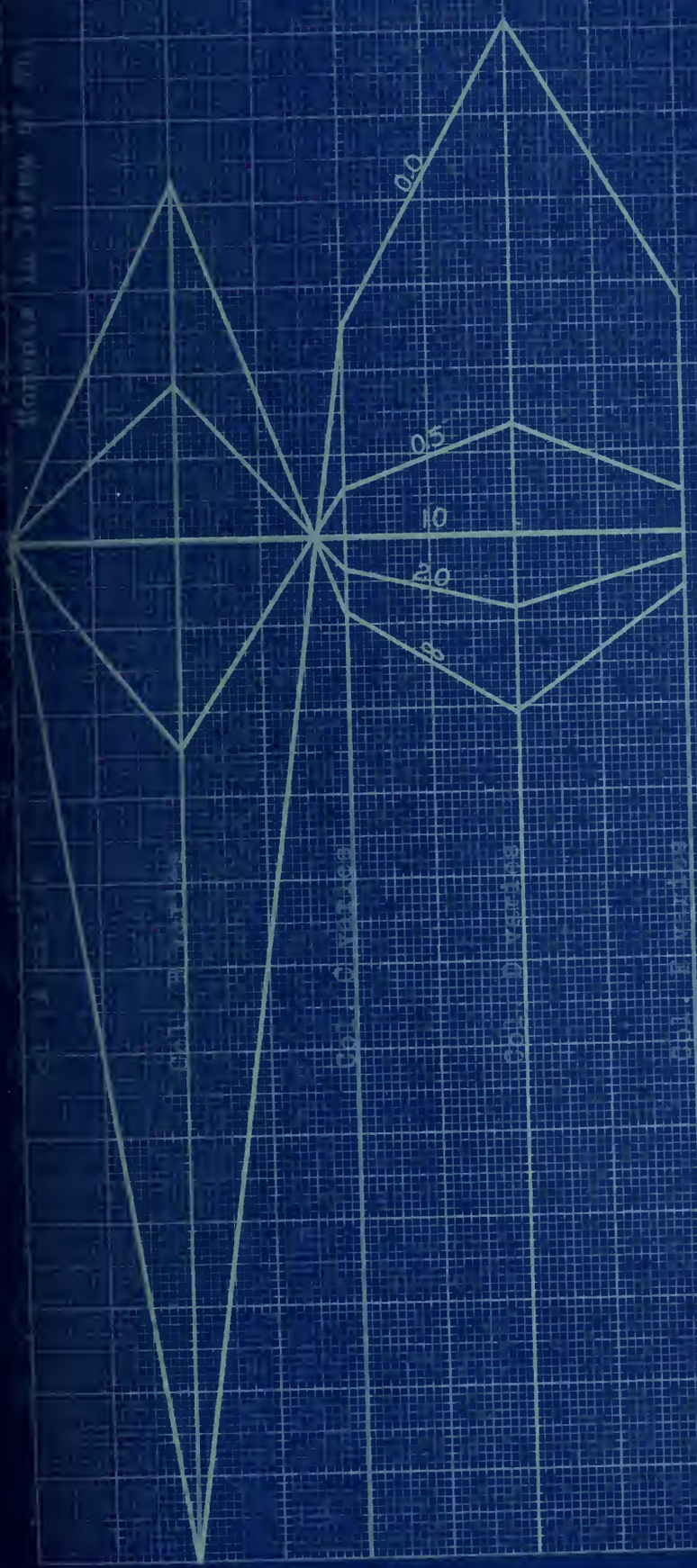

Diagram 5C.

FOUR SPAN BENT

Moments in Column B.

K of One Column Changes.
K of Other Columns Unity.
K of Girder Unity.
Numbers on Curves Indicate magnitudes of K for member which varies.

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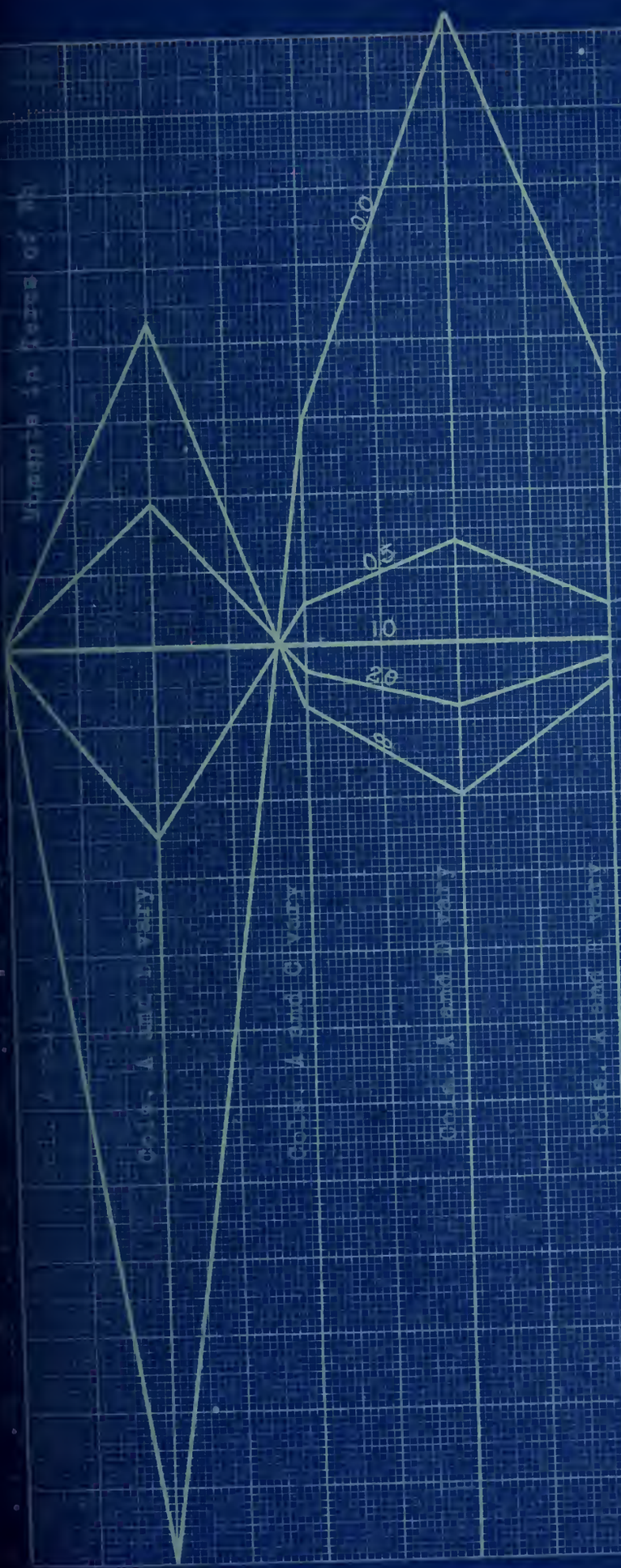

Diagram 51.

FOUR-SPAN BENT

Moments in Column B

X's of Column A and One Other Column Change Simultaneous

X's of All Other Columns Unity
X's of All Girders Unity.
Numbers on Curves Indicate Magnitudes of X in Members Which Vary.

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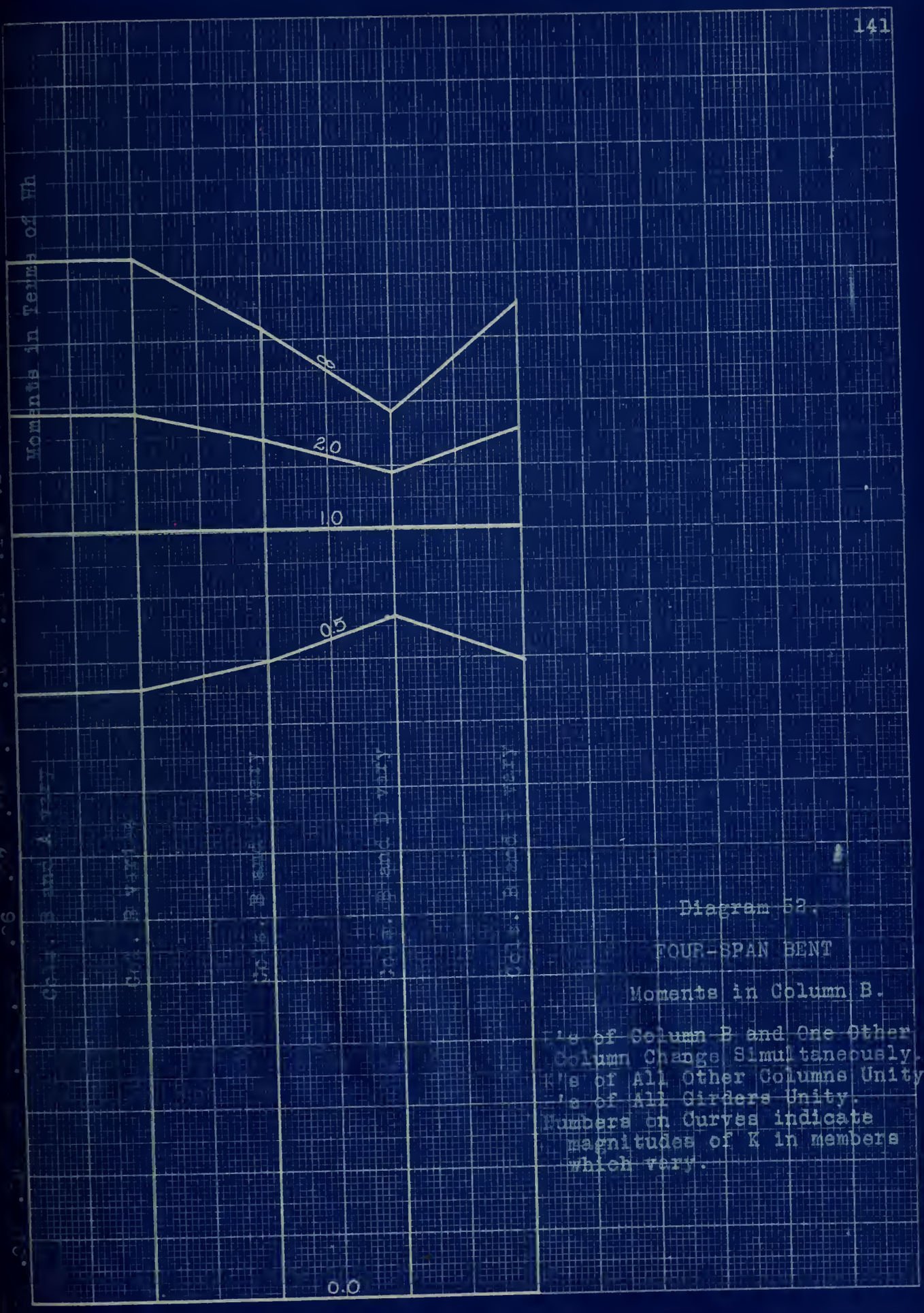


Diagram 52.

FOUR-SPAN BENT

Moments in Column B.

K's of Column B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

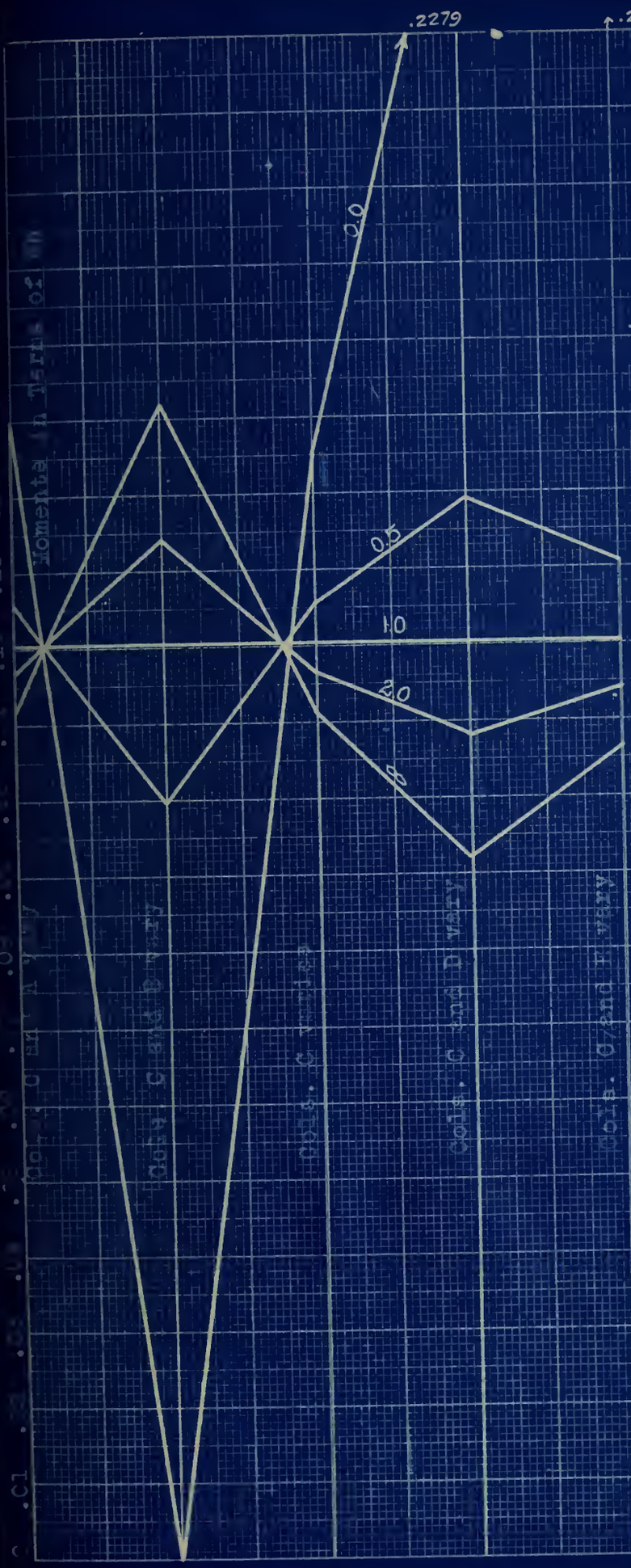


Diagram 56.

FOUR-SPAN BENT

Moments in Column B.

K's of Column C and One Other Column Change Simultaneously.

K's of All Other Columns Unity

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary.

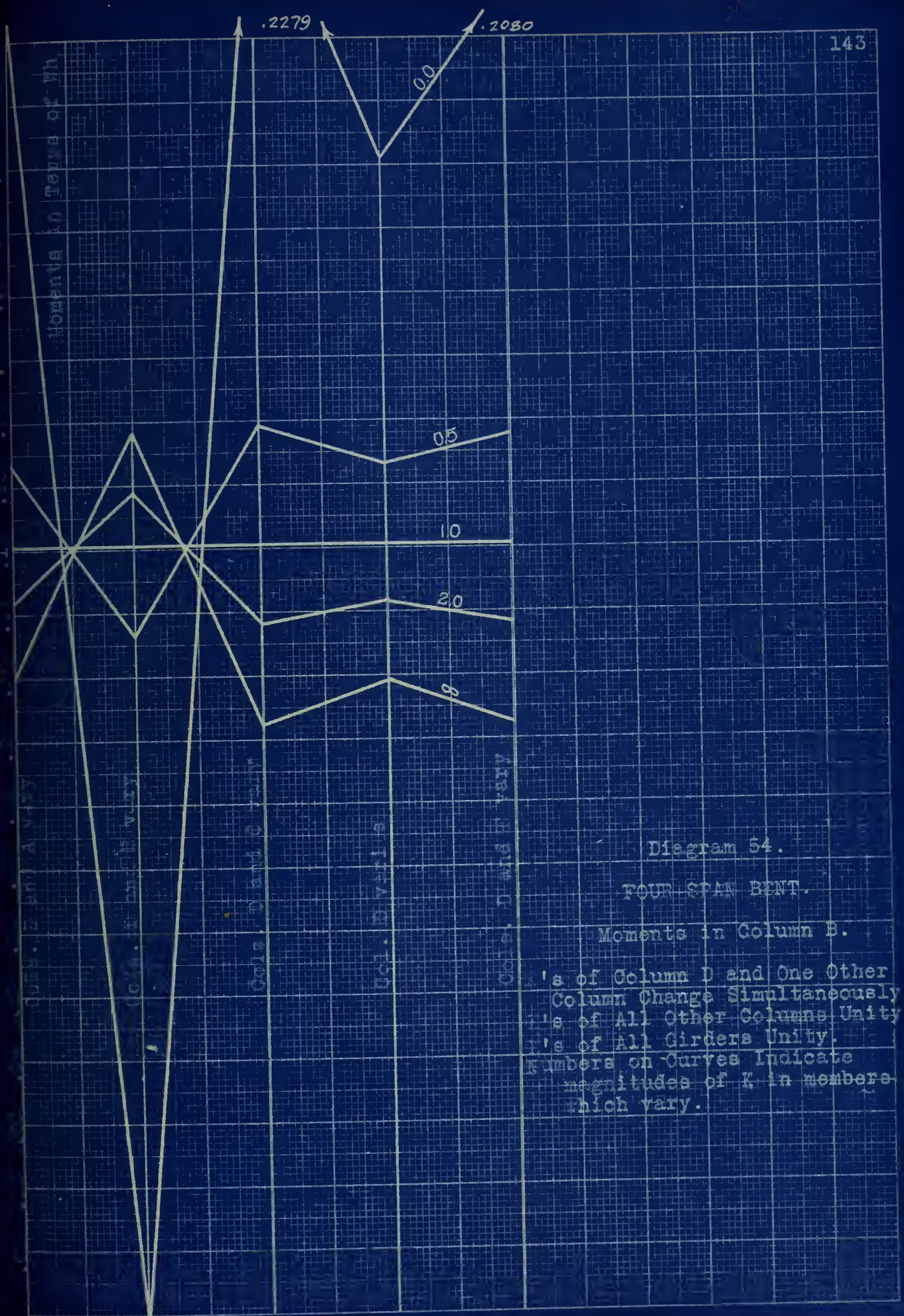


Diagram 54.

FOUR SPAN BENT.

Moments in Column B.

's of Column D and One Other Column Change Simultaneously
's of All Other Columns Unity
's of All Girders Unity.
Numbers on Curves Indicate magnitudes of K in members which vary.

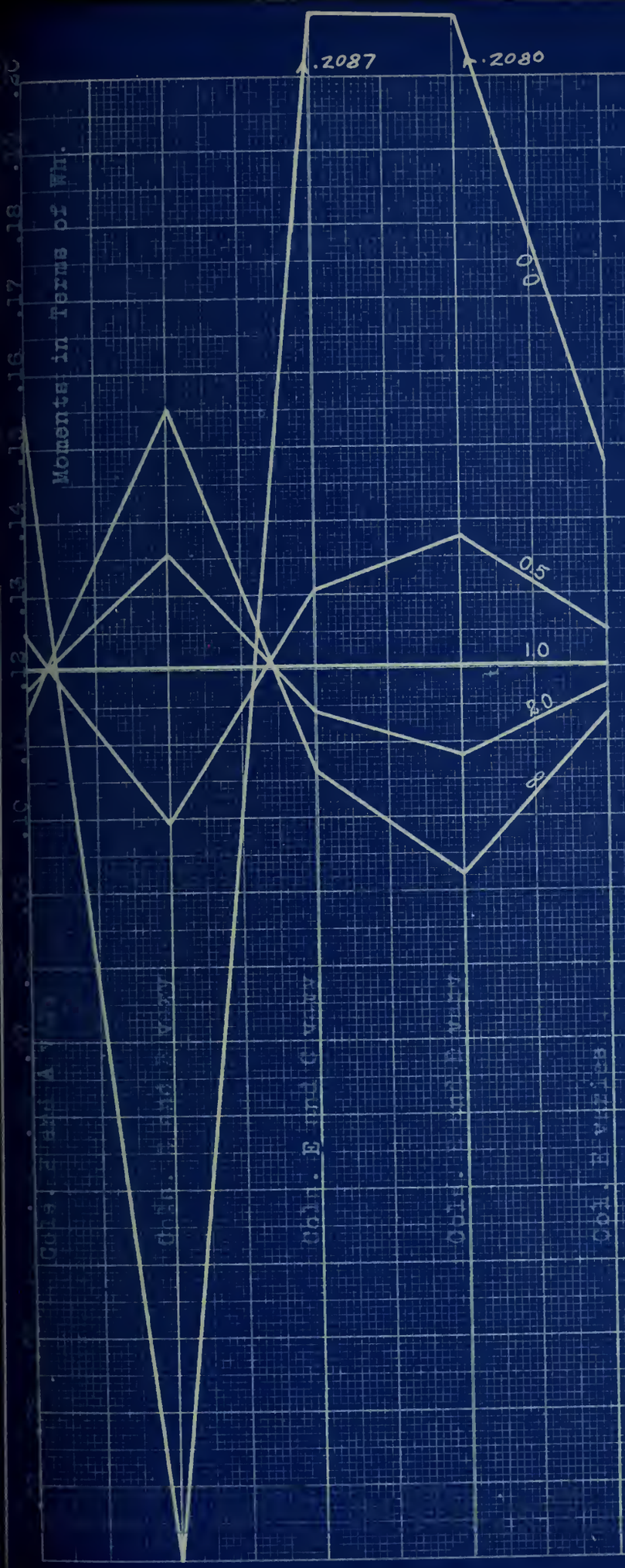


Diagram 55.

FOUR-SPAN BENT

Moments in Column B

K's of Column F and One Other Column Change Simultaneously; K's of All Other Columns Unity; K's of All Girders Unity. Numbers on Curves Indicate Magnitudes of K in Members which Vary.

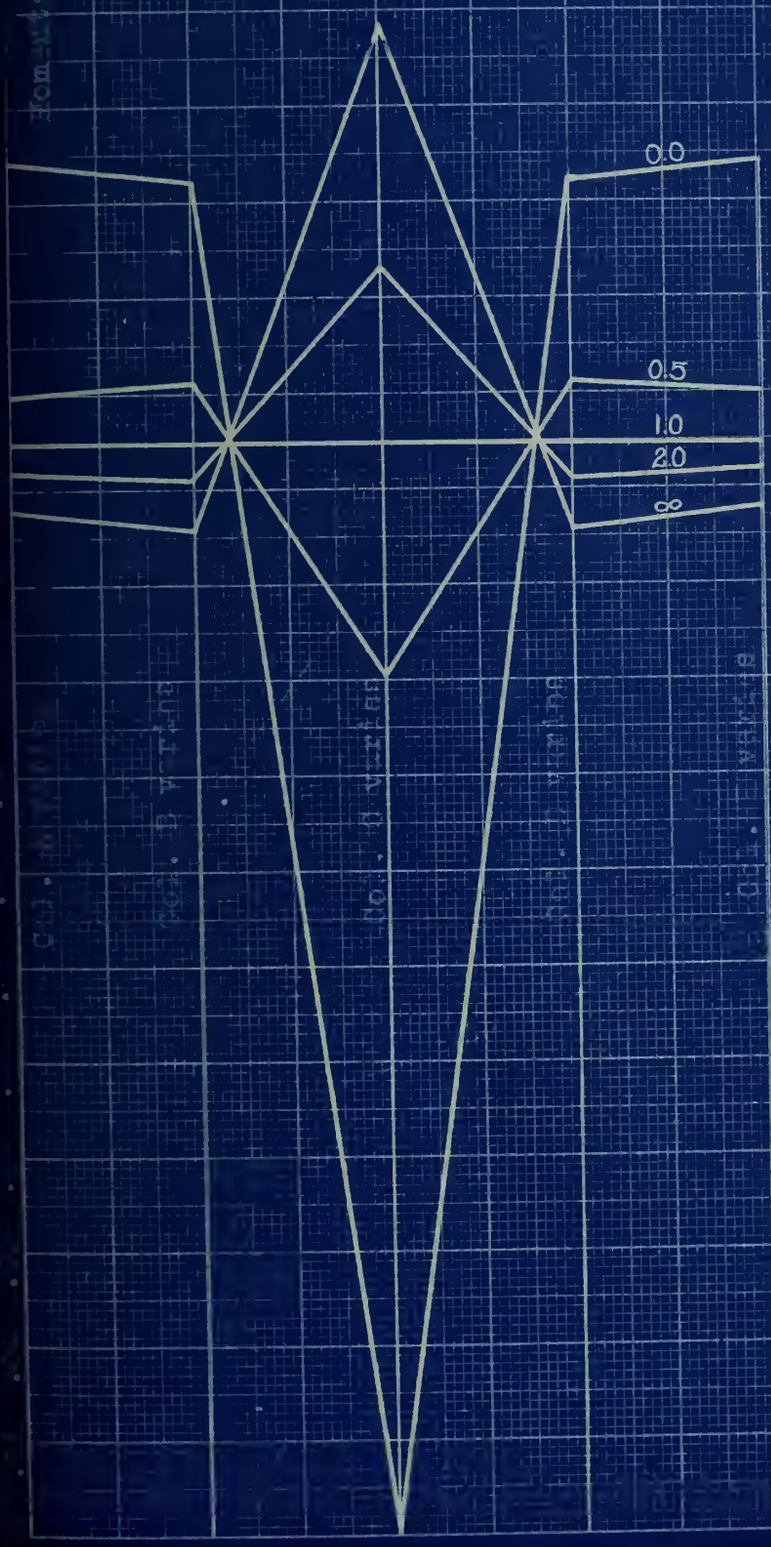


Diagram 56.

FOUR-SPAN BENT

Moments in Column C.

K of one Column Changes.
K's of Other Columns Unity.
K of Girder Unity.
Numbers on Curves indicate magnitudes of K for member which varies.

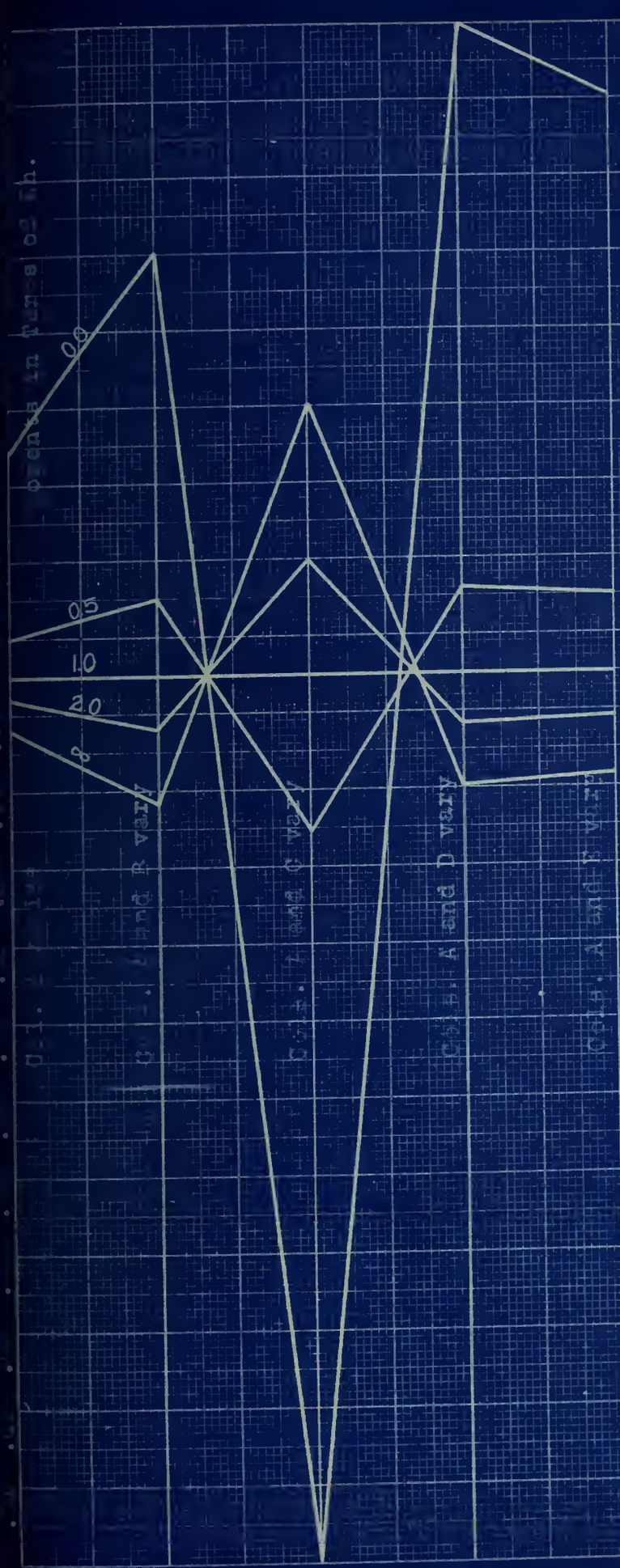
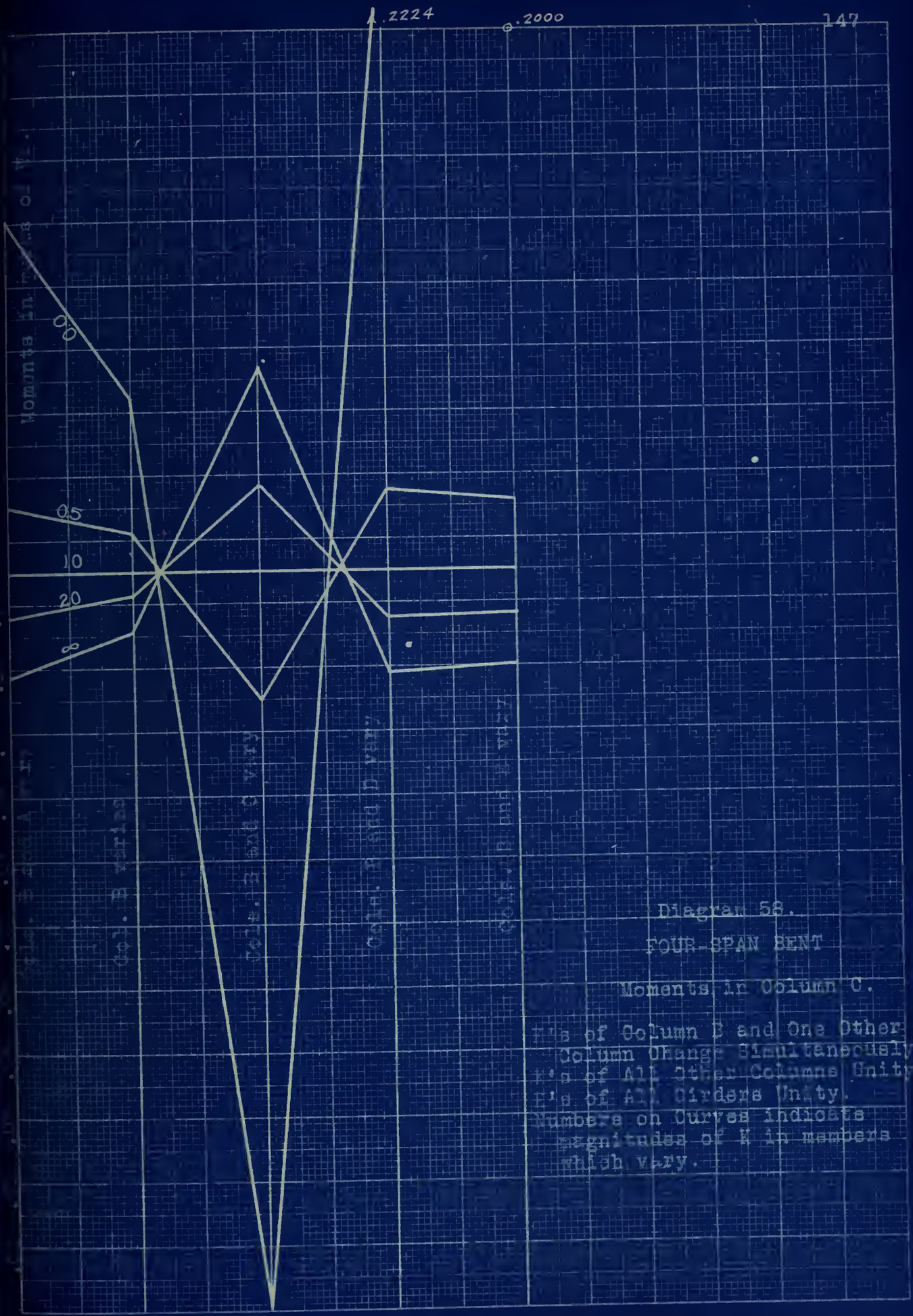


Diagram 57.

FOUR-SPAN BENT.

Momentsin Column C

K's of Colum A and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.



Lloyde's University Stacks

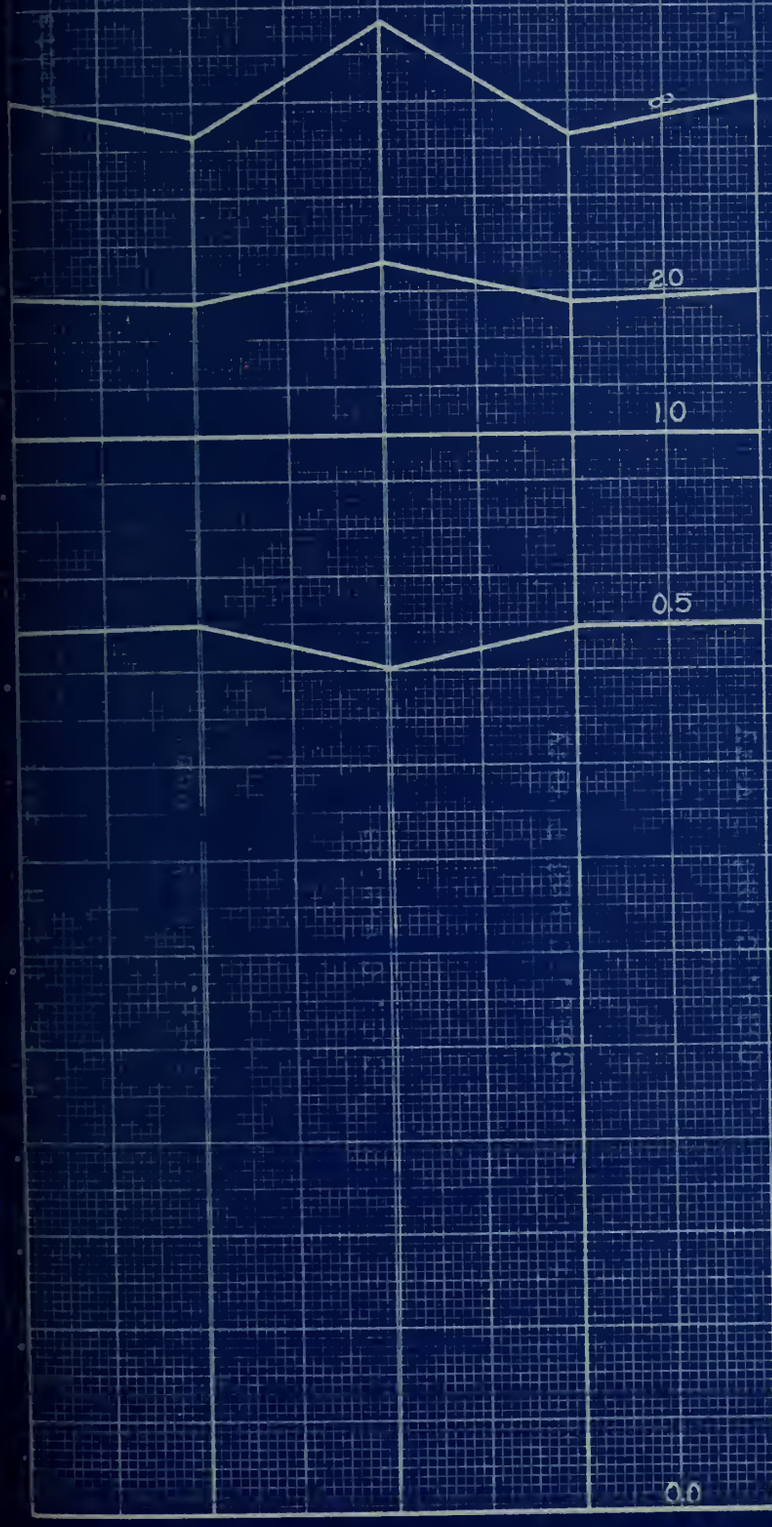


Diagram 39.

FOUR-SPAN BENT

Moments in Column C.

K's of Column C and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

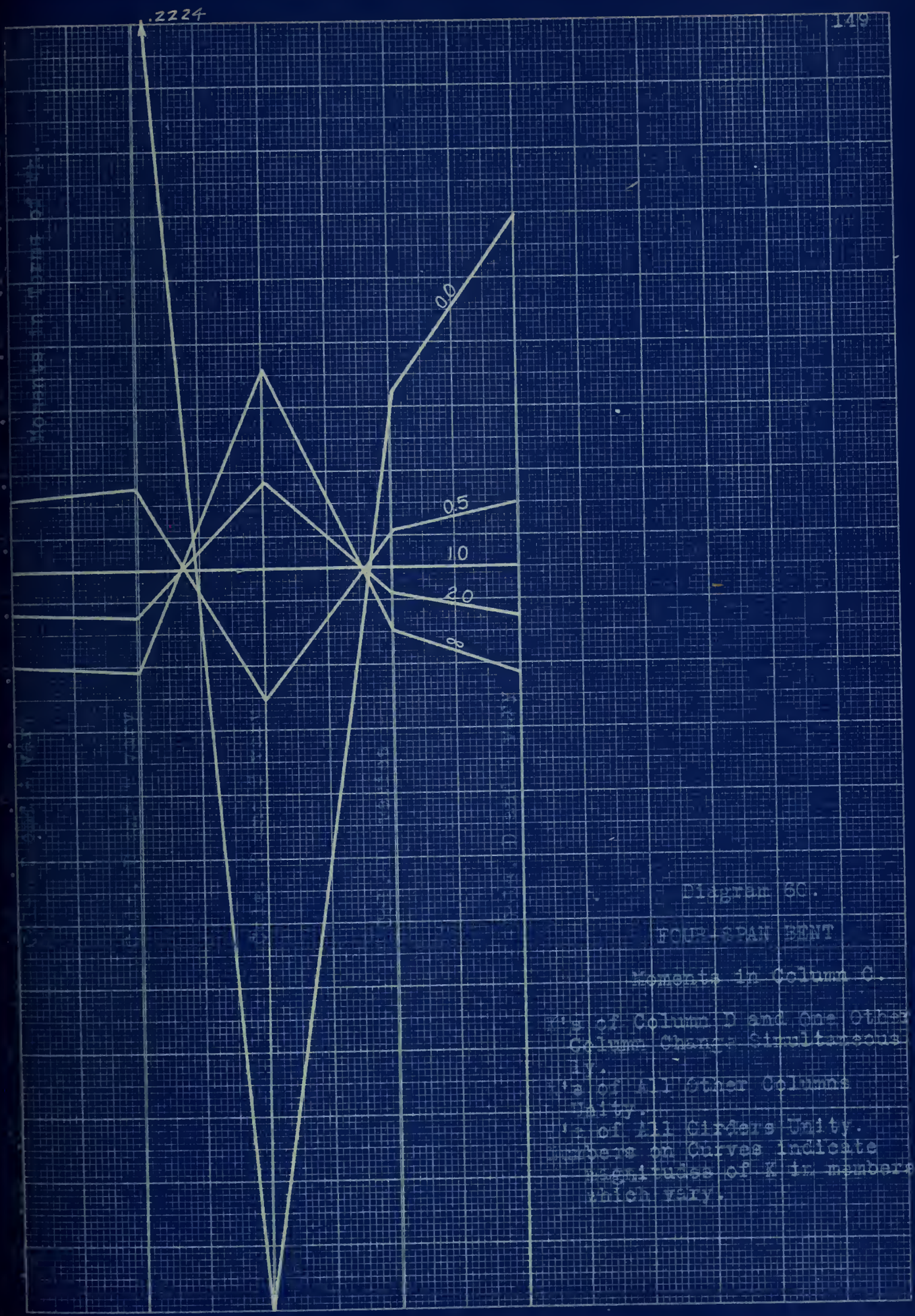


Diagram 66.

FOUR-SPAN BENT

Moments in Column C.

K of Column D and one Other Column Chang - Simultaneously.

K's of All Other Columns Unity.

K of All Girders Unity.

Numbers on Curves indicate Magnitudes of K in members which vary.

Lloyde's University Store

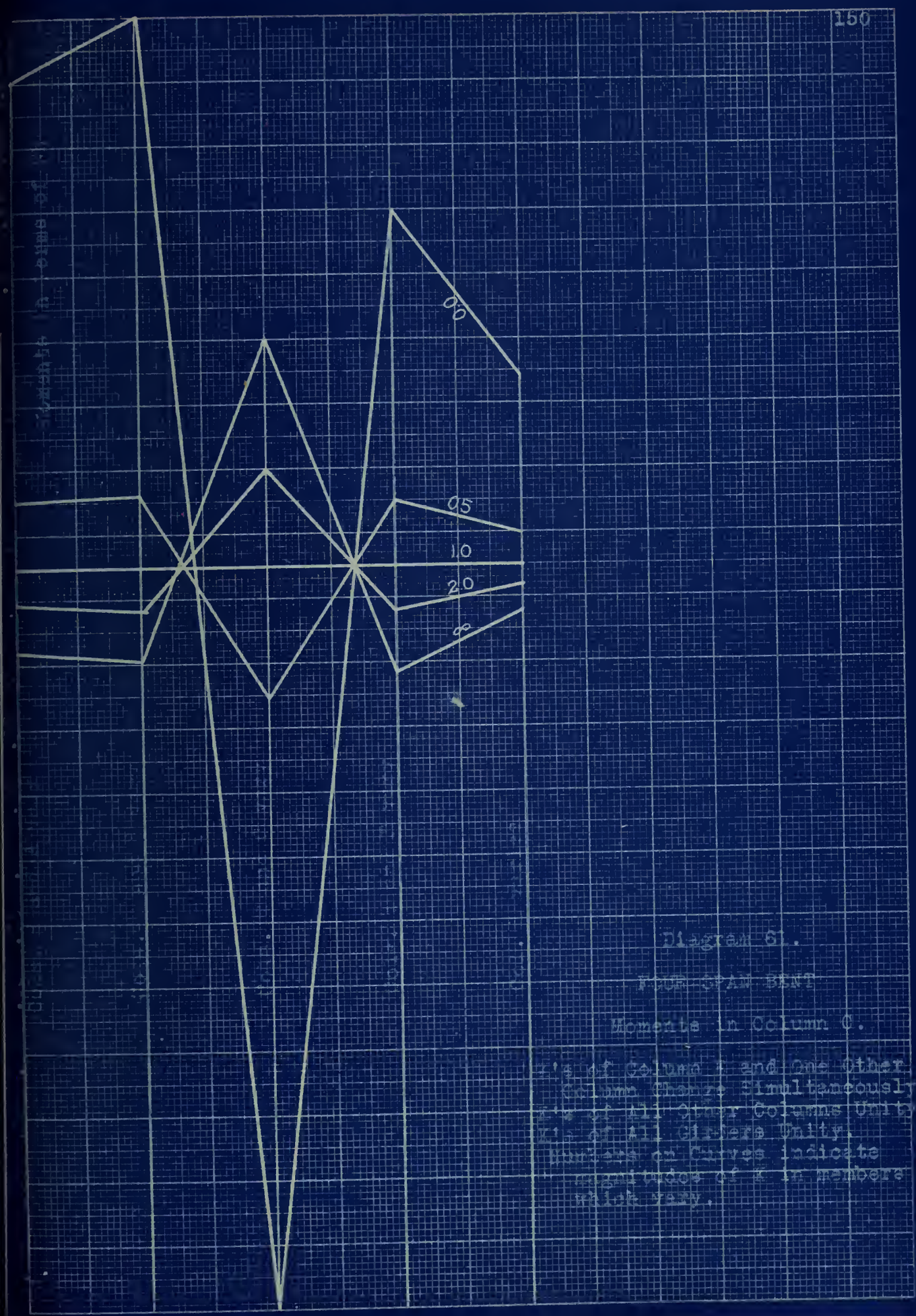


Diagram 61.

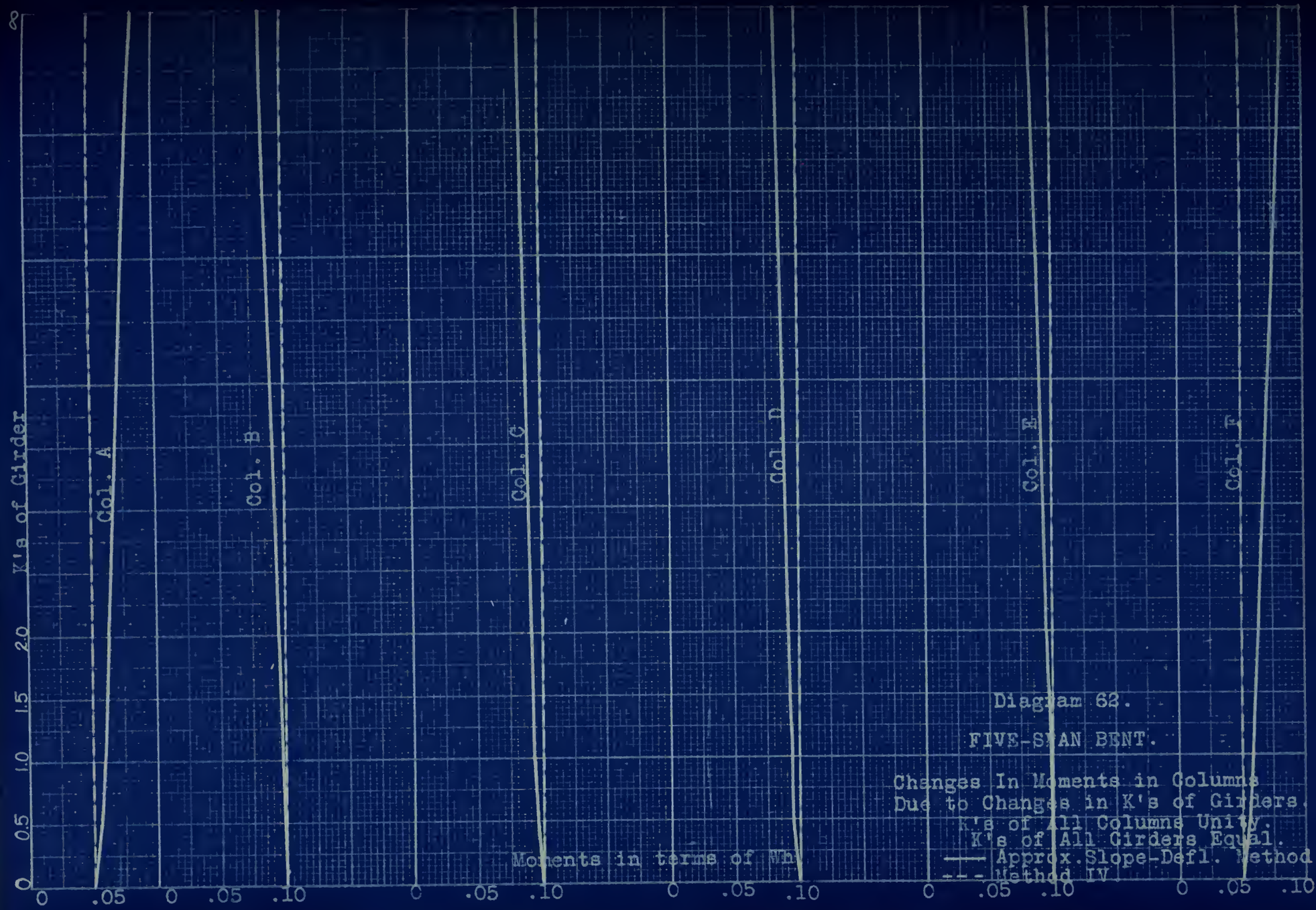
FRAME-BENT

Moments in Column C.

K's of Column I and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

FIVE-SPAN BENT.

Diagrams 62 to 150, inclusive.


K's of Girder
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
0
.05
0
.05
.10
0
.05
.10
0
.05
.10
0
.05
.10
0
.05
.10
Moments in terms of Wh
Diagram 62.
FIVE-SPAN BENT.
Changes In Moments in Columns
Due to Changes in K's of Girders.
K's of All Columns Unity.
K's of All Girders Equal.
—— Approx.Slope-Defl. Method
- - - Method IV

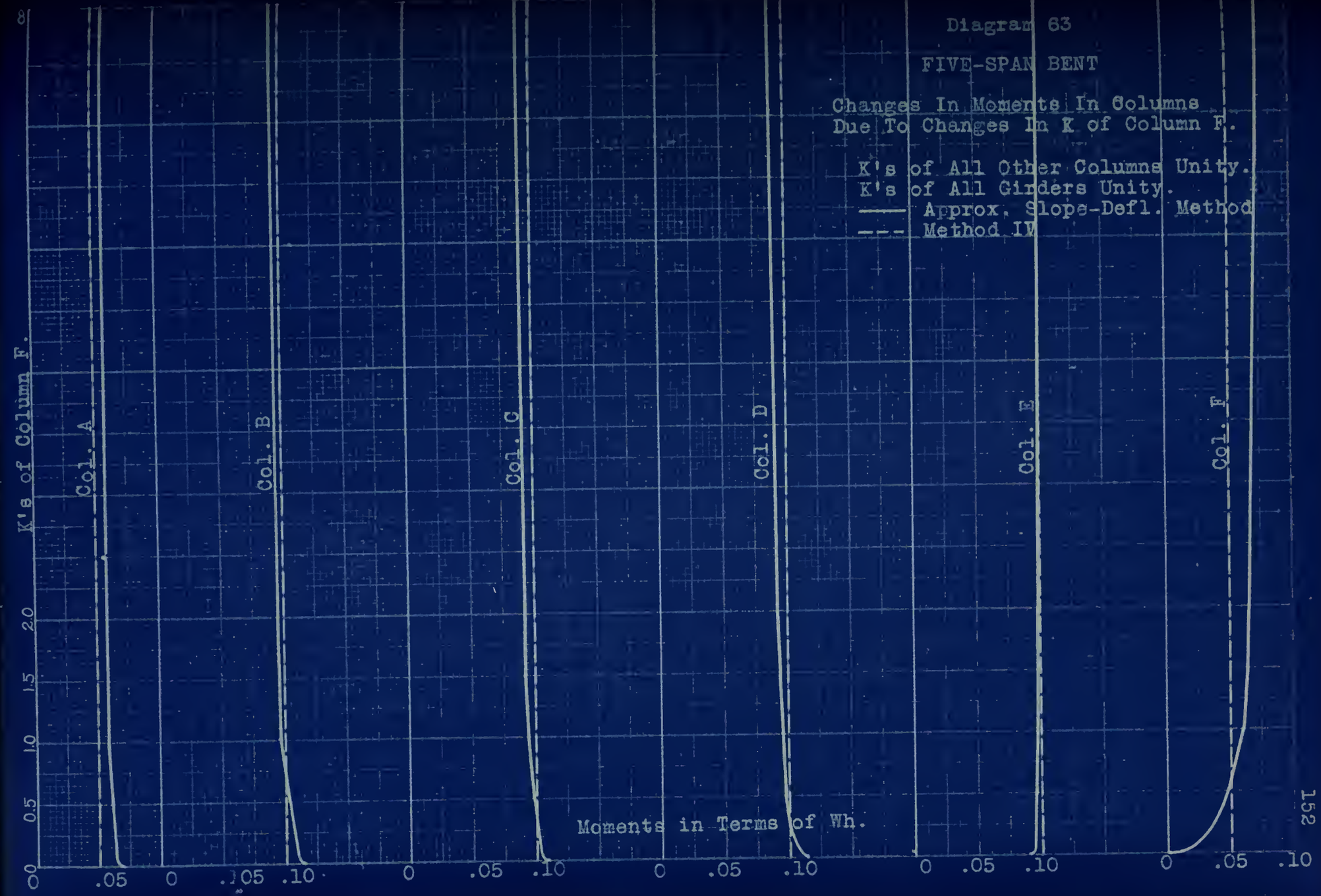


Diagram 63
FIVE-SPAN BENT
Changes In Moments In Columns
Due To Changes In K of Column F.
K's of All Other Columns Unity.
K's of All Girders Unity.
—— Approx. Slope-Defl. Method
--- Method IV
K's of Column F.
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh.
0
0.5
1.0
1.5
2.0
.05
.10

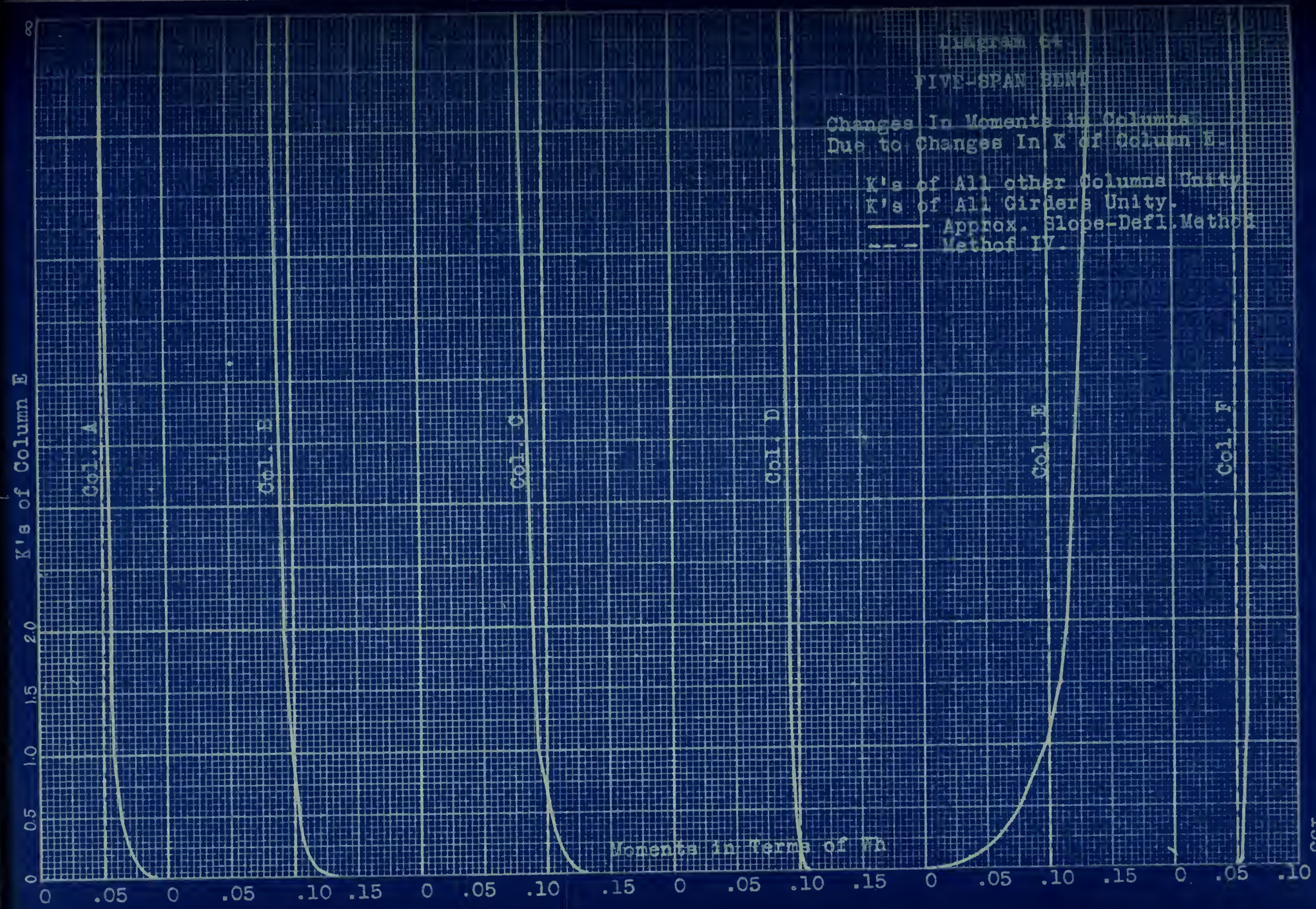


Diagram 64
FIVE-SPAN BENT
Changes In Moments In Columns
Due to Changes In K of Column E
K's of All other Columns Unity.
K's of All Girders Unity.
Approx. Slope-Defl. Method
Method IV.
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
K's of Column E
0
0.5
1.0
1.5
2.0
Moments in Terms of Fh
0
.05
.10
.15

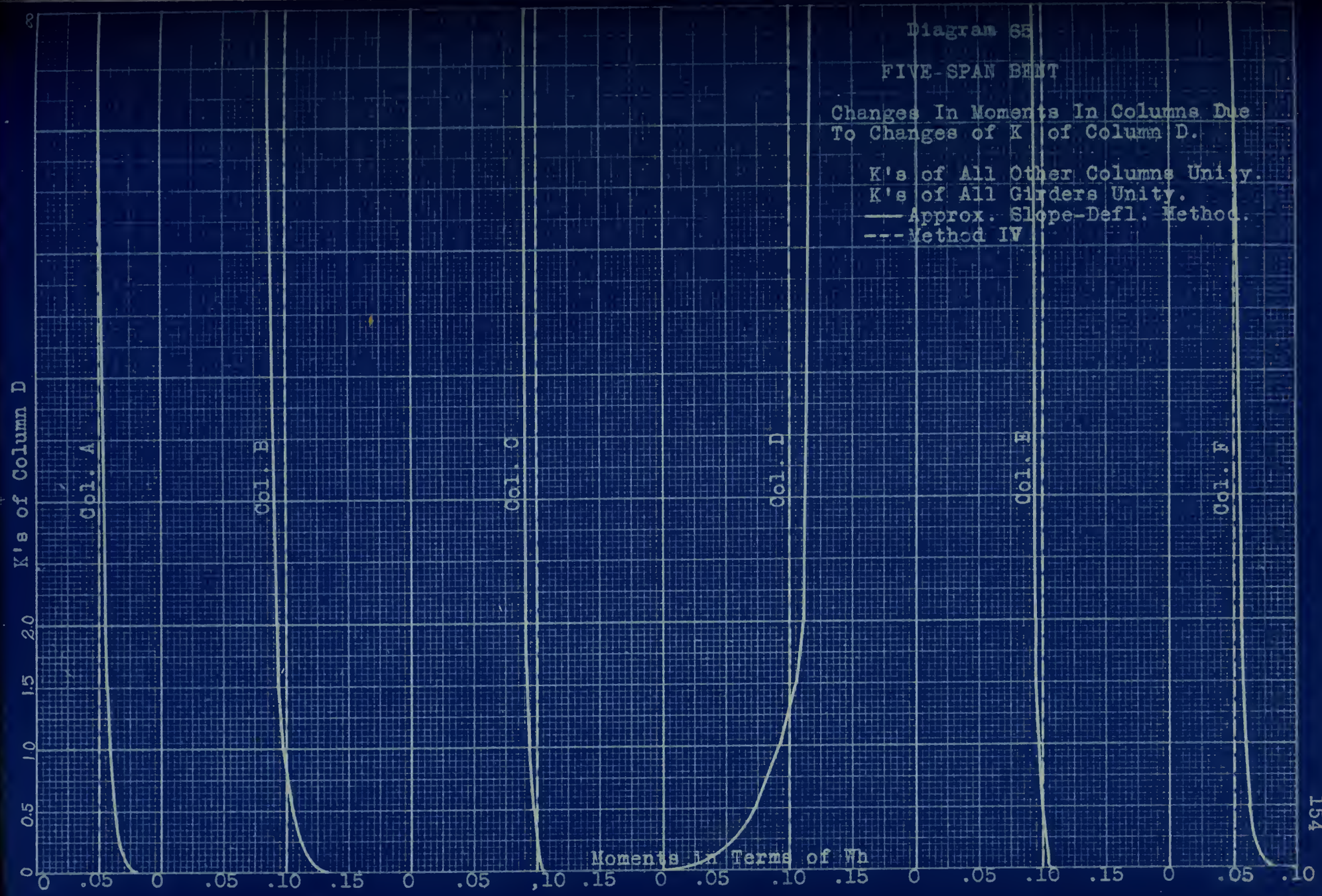


Diagram 65
FIVE-SPAN BENT
Changes In Moments In Columns Due To Changes of K of Column D.
K's of All Other Columns Unity.
K's of All Girders Unity.
——Approx. Slope-Defl. Method.
---Method IV
K's of Column D
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments In Terms of Wh
0 .05 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10

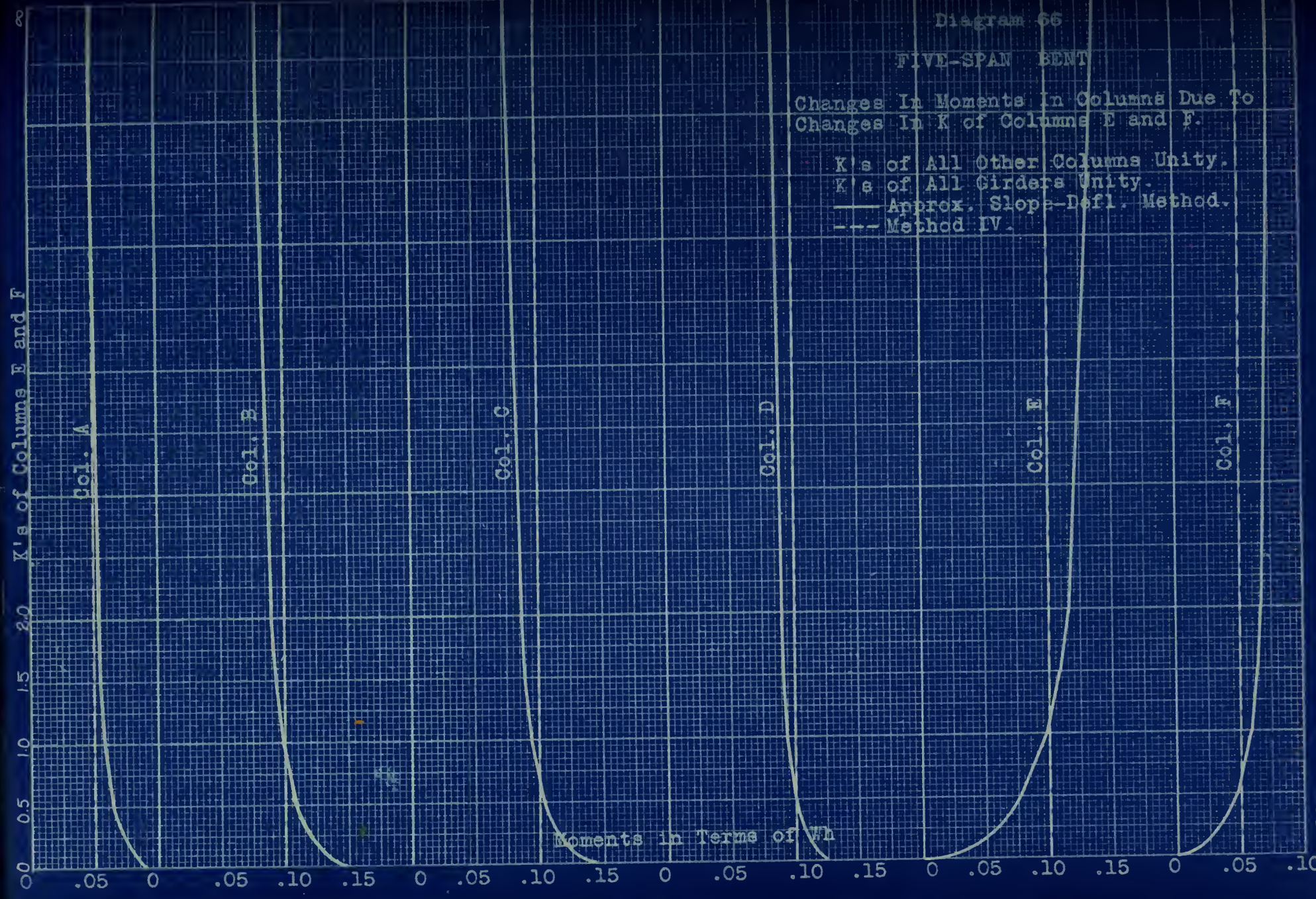


Diagram 66
FIVE-SPAN BENT
Changes In Moments In Columns Due To Changes In K of Columns E and F.
K's of All Other Columns Unity.
K's of All Girders Unity.
—— Approx. Slope-Defl. Method.
- - - Method IV.
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
K's of Columns E and F
Moments in Terms of Wh
0
0.5
1.0
1.5
2.0
.05
.10
.15

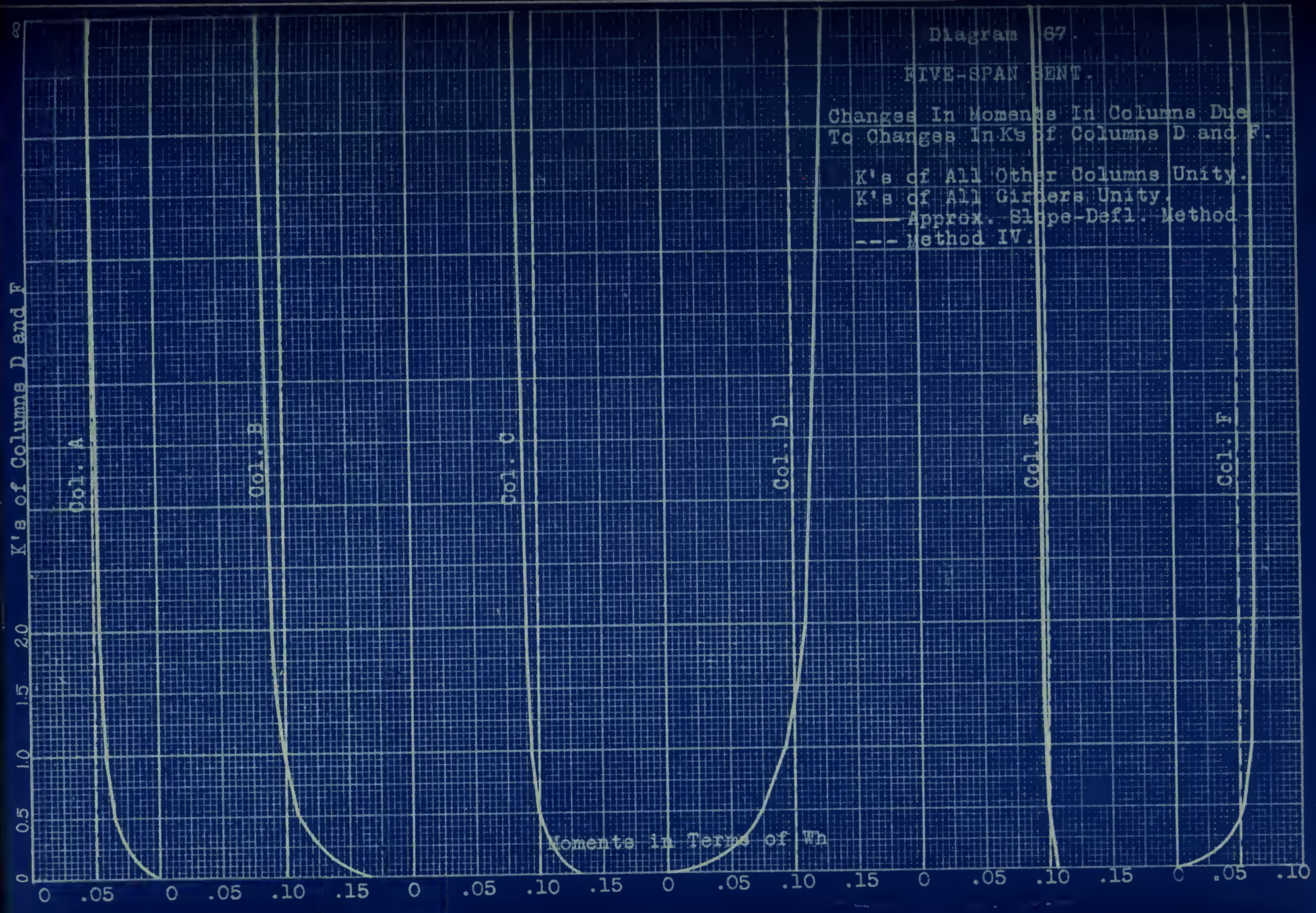


Diagram 67.
FIVE-SPAN BENT.
Changes In Moments In Columns Due To Changes In Ks of Columns D and F.
K's of All Other Columns Unity.
K's of All Girders Unity.
—— Approx. Slope-Defl. Method
--- Method IV.
K's of Columns D and F
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
0
0.5
1.0
1.5
2.0
0
.05
.10
.15

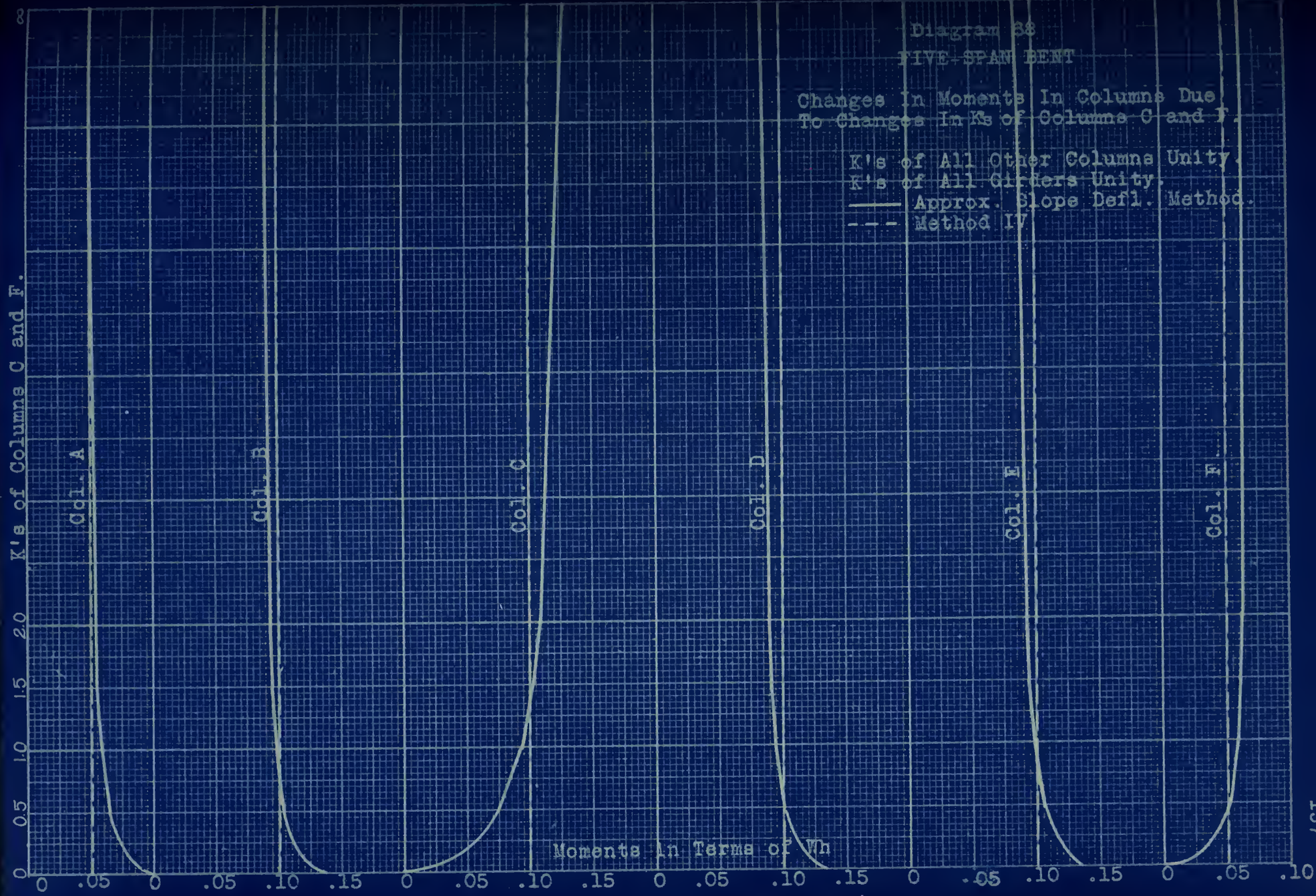


Diagram 68
FIVE-SPAN BENT
Changes In Moments In Columns Due To Changes In Ks of Columns C and F.
K's of All Other Columns Unity.
K's of All Girders Unity.
Approx. Slope Defl. Method.
Method IV
K's of Columns C and F.
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments In Terms of Wh
0
.05
.10
.15

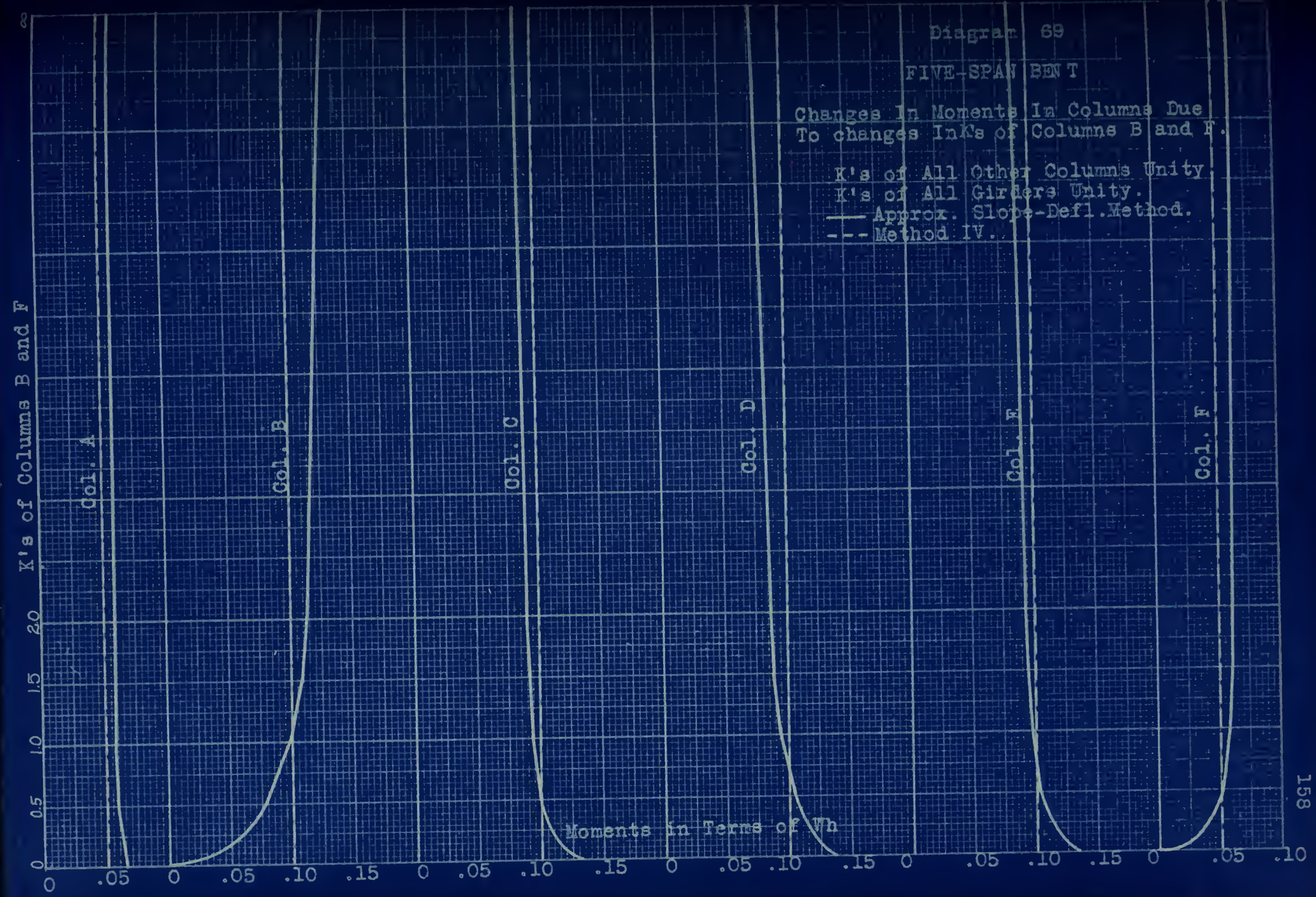


Diagram 69
FIVE-SPAN BENT
Changes In Moments In Columns Due
To changes In K's of Columns B and F.
K's of All Other Columns Unity
K's of All Girders Unity.
—— Approx. Slope-Defl. Method.
- - - Method IV.
K's of Columns B and F
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
0
.05
.10
.15

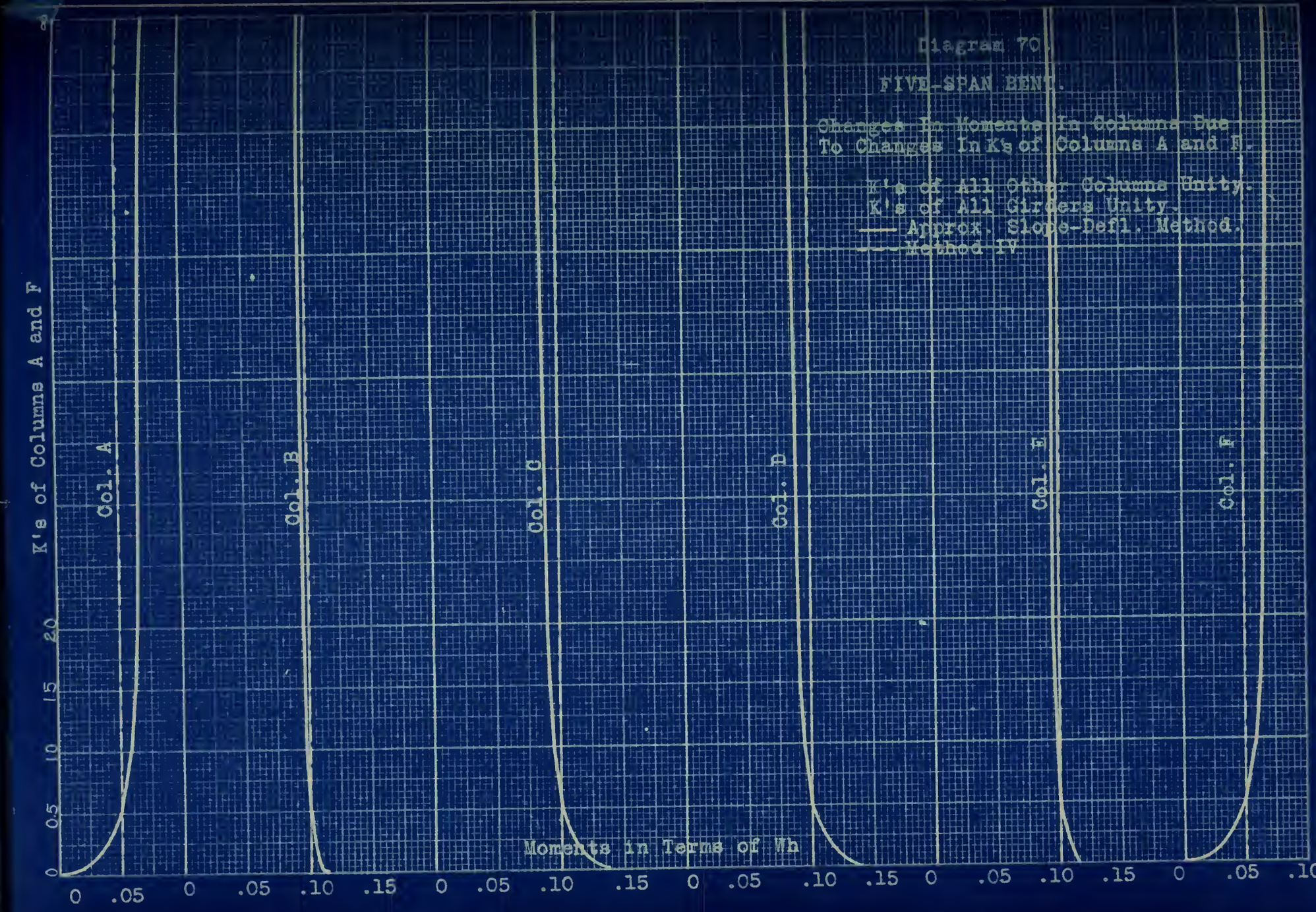


Diagram 70
FIVE-SPAN BENT.
Changes In Moments In Columns Due
To Changes In Ks of Columns A and F.
K's of All Other Columns Unity.
K's of All Girders Unity.
—— Approx. Slope-Defl. Method.
—— Method IV.
K's of Columns A and F
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
0
0.5
1.0
1.5
2.0
0
.05
.10
.15

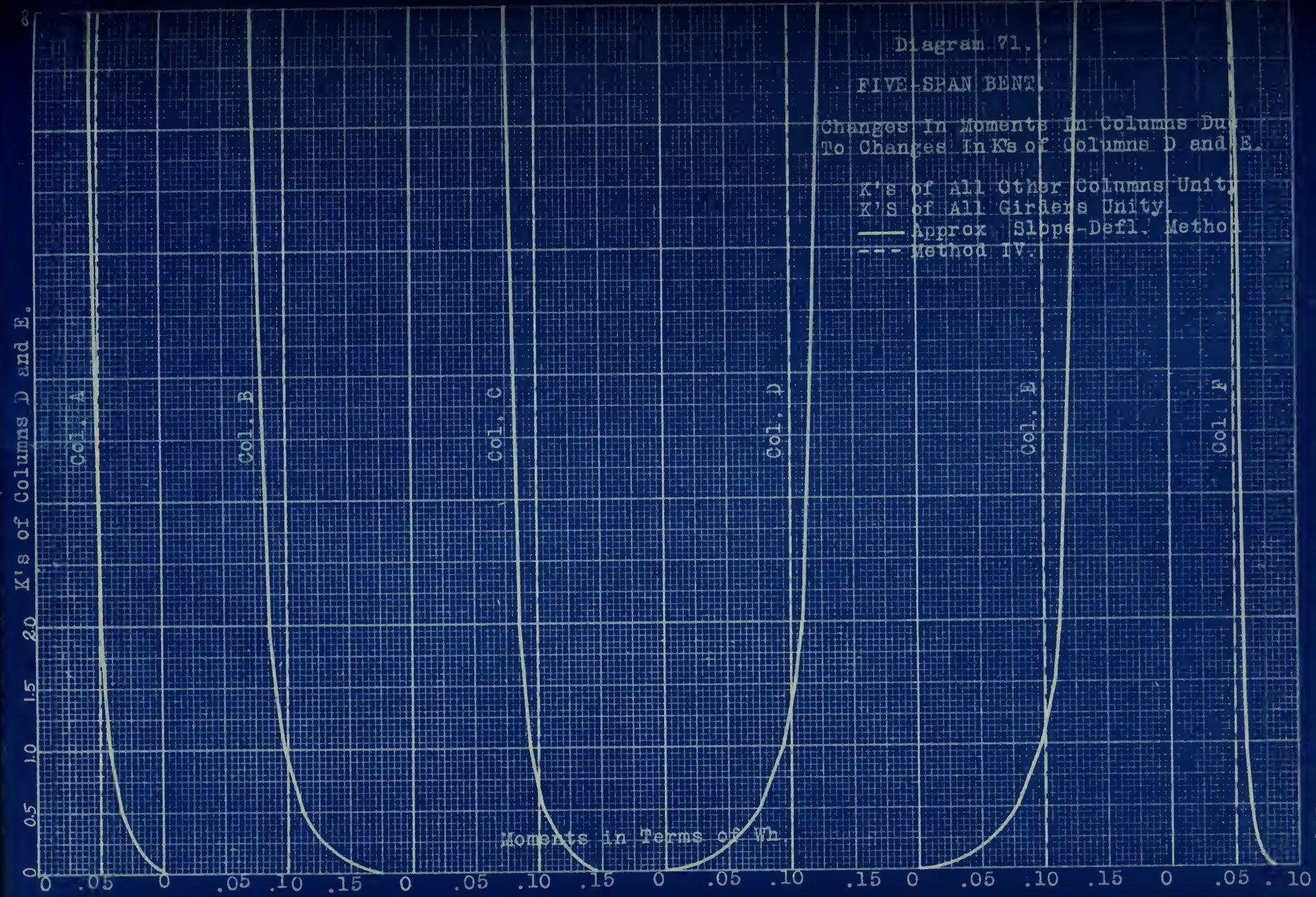


Diagram 71.
FIVE-SPAN BENT.
Changes In Moments In Columns Due
To Changes In Ks of Columns D and E.
K's of All Other Columns Unity
K'S of All Girders Unity.
—— Approx Slope-Defl. Method
--- Method IV.
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh.
K's of Columns D and E.
0
0.5
1.0
1.5
2.0
0 .05 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10

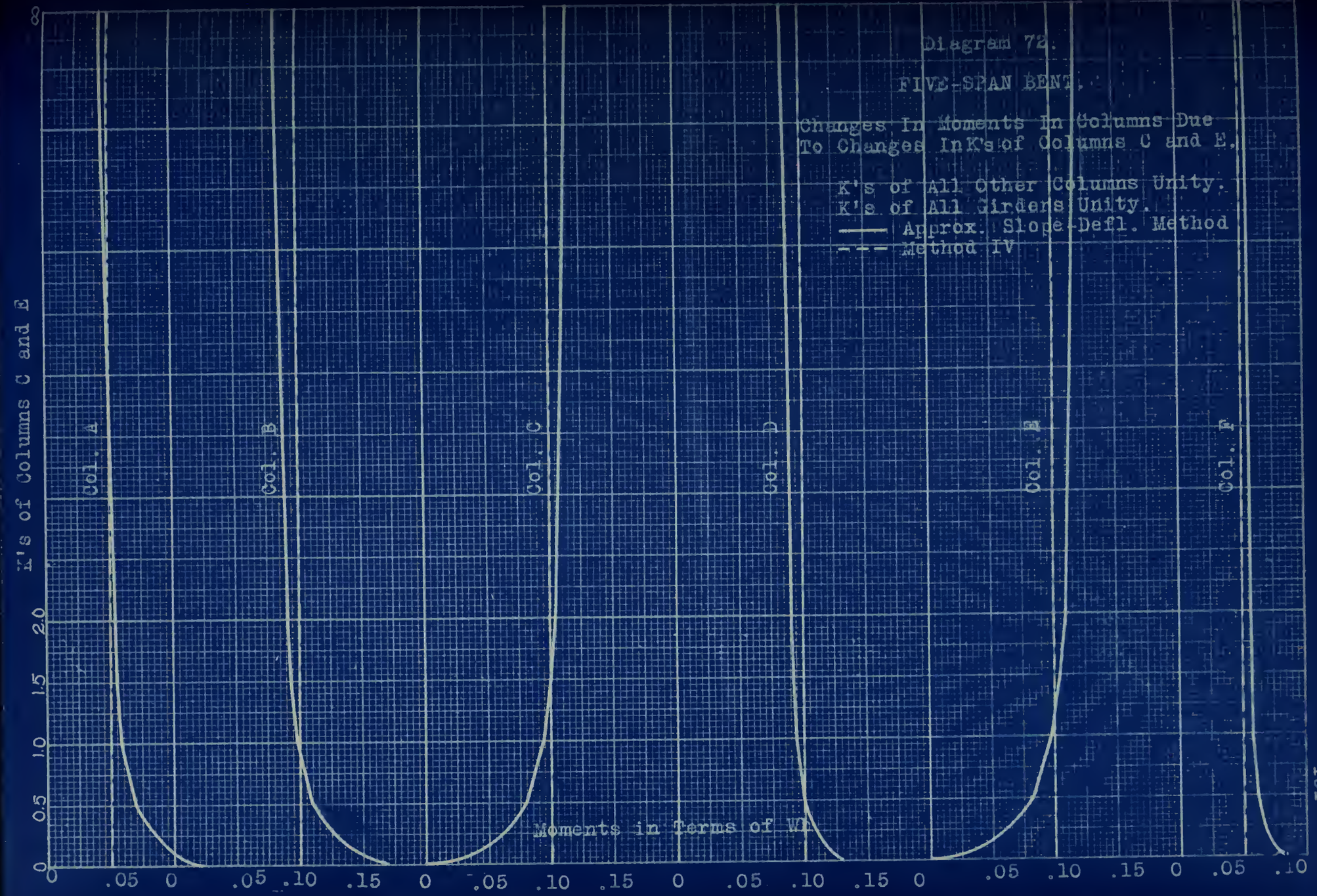


Diagram 72.
FIVE-SPAN BENT.
Changes In Moments In Columns Due To Changes In K's of Columns C and E.
K's of All Other Columns Unity.
K's of All Girders Unity.
Approx. Slope-Defl. Method
Method IV
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
K's of Columns C and E
0
0.5
1.0
1.5
2.0
Moments in Terms of WL
0
.05
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10

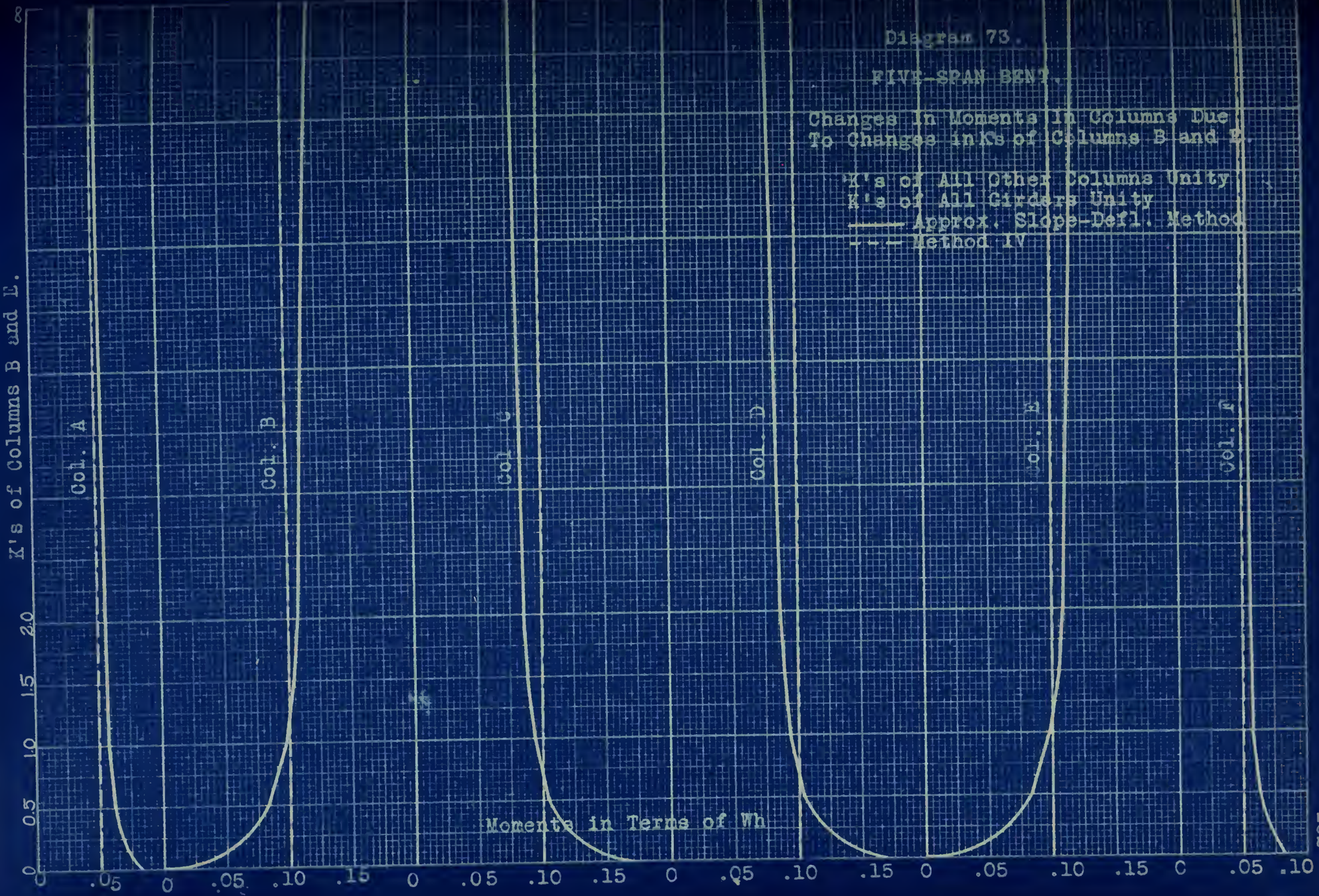


Diagram 73.
FIVE-SPAN BENT.
Changes In Moments In Columns Due To Changes in Ks of Columns B and E.
K's of All Other Columns Unity
K's of All Girders Unity
Approx. Slope-Defl. Method
Method IV
K's of Columns B and E.
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
0
.05
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10

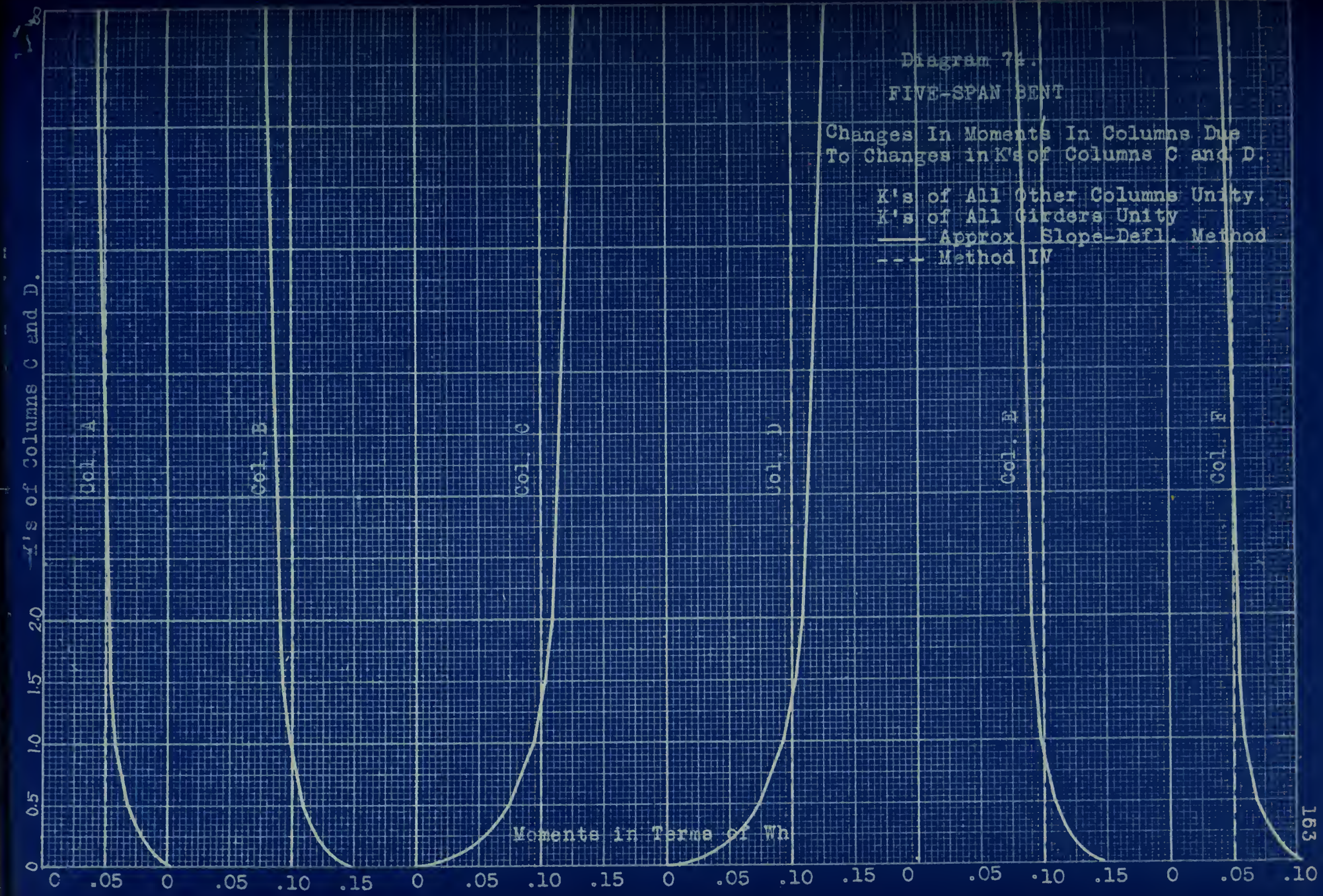


Diagram 74.
FIVE-SPAN BENT
Changes In Moments In Columns Due To Changes in K's of Columns C and D.
K's of All Other Columns Unity.
K's of All Girders Unity
Approx. Slope-Defl. Method
Method IV
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
K's of Columns C and D.
0
0.5
1.0
1.5
2.0
Moments in Terms of Wh
C .05 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10

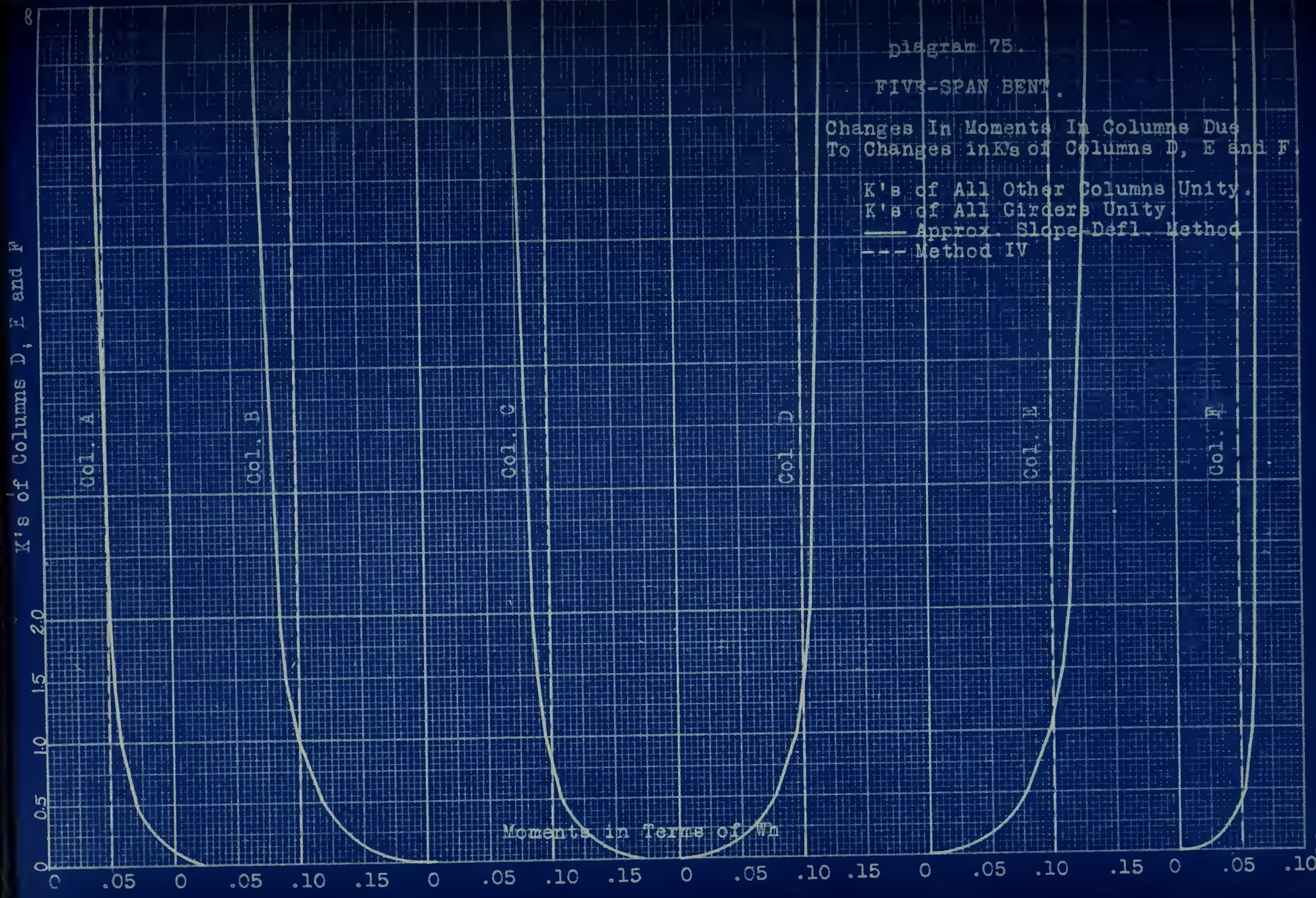


Diagram 75.
FIVE-SPAN BENT.
Changes In Moments In Columns Due To Changes in K's of Columns D, E and F.
K's of All Other Columns Unity.
K's of All Girders Unity
—— Approx. Slope-Defl. Method
--- Method IV
K's of Columns D, E and F
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wn
0
0.5
1.0
1.5
2.0
0
.05
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10

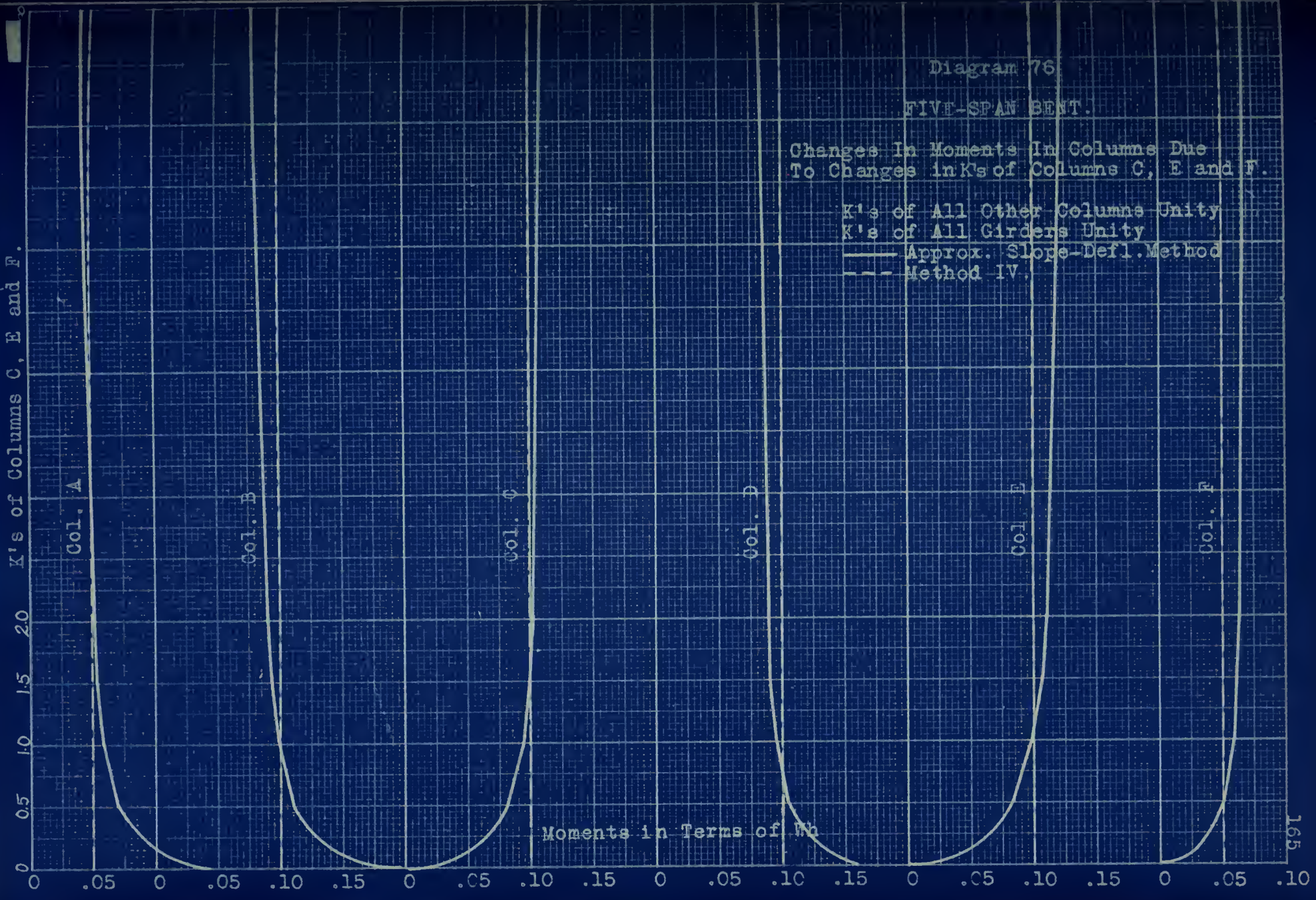


Diagram 76
FIVE-SPAN BENT.
Changes In Moments In Columns Due
To Changes in K's of Columns C, E and F.
K's of All Other Columns Unity
K's of All Girders Unity
Approx. Slope-Defl. Method
Method IV.
K's of Columns C, E and F.
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
0
.05
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10

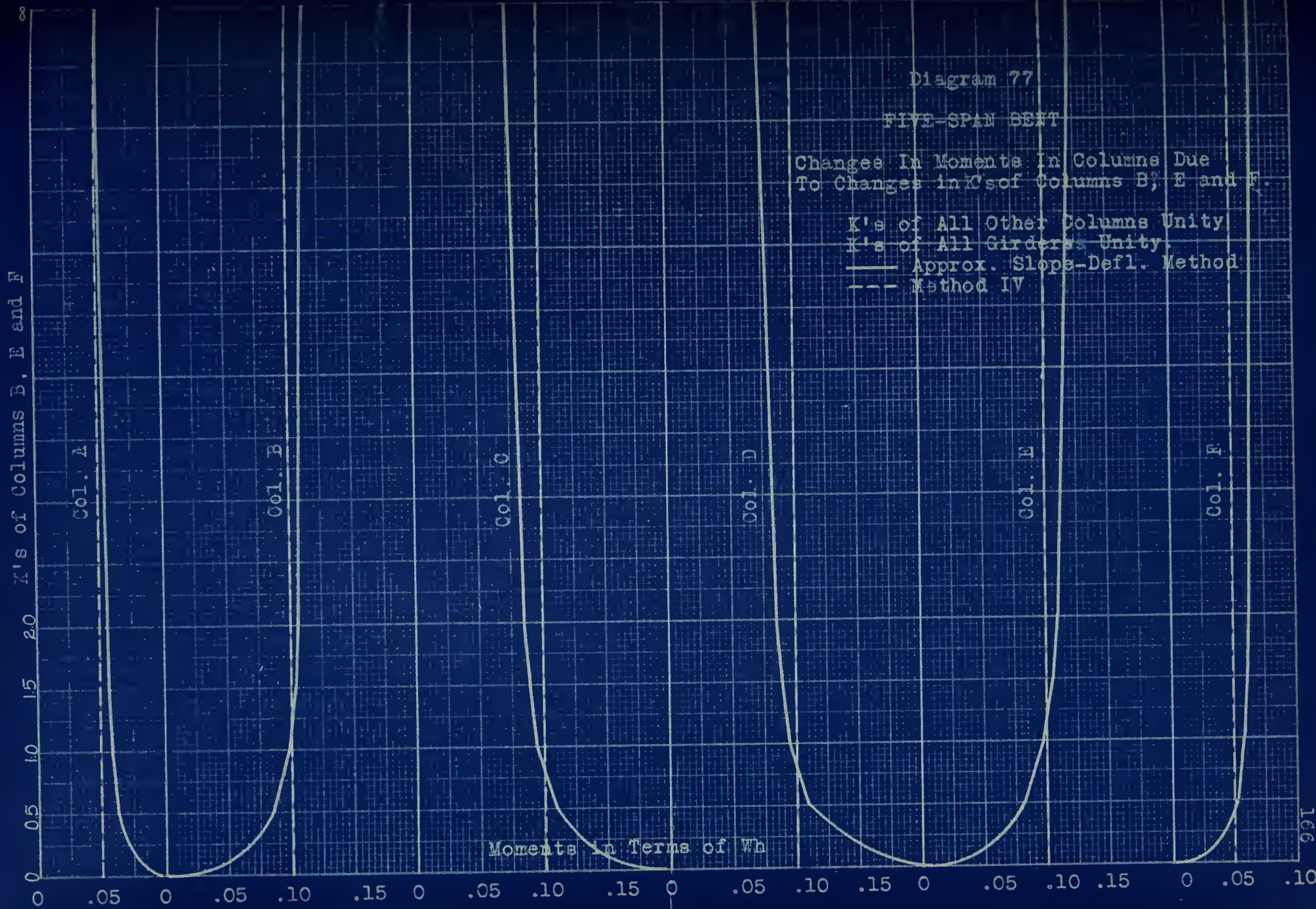


Diagram 77
FIVE-SPAN BENT
Changes In Moments In Columns Due To Changes in K's of Columns B, E and F.
K's of All Other Columns Unity
K's of All Girders Unity
Approx. Slope-Defl. Method
Method IV
K's of Columns B, E and F
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
0
.05
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10

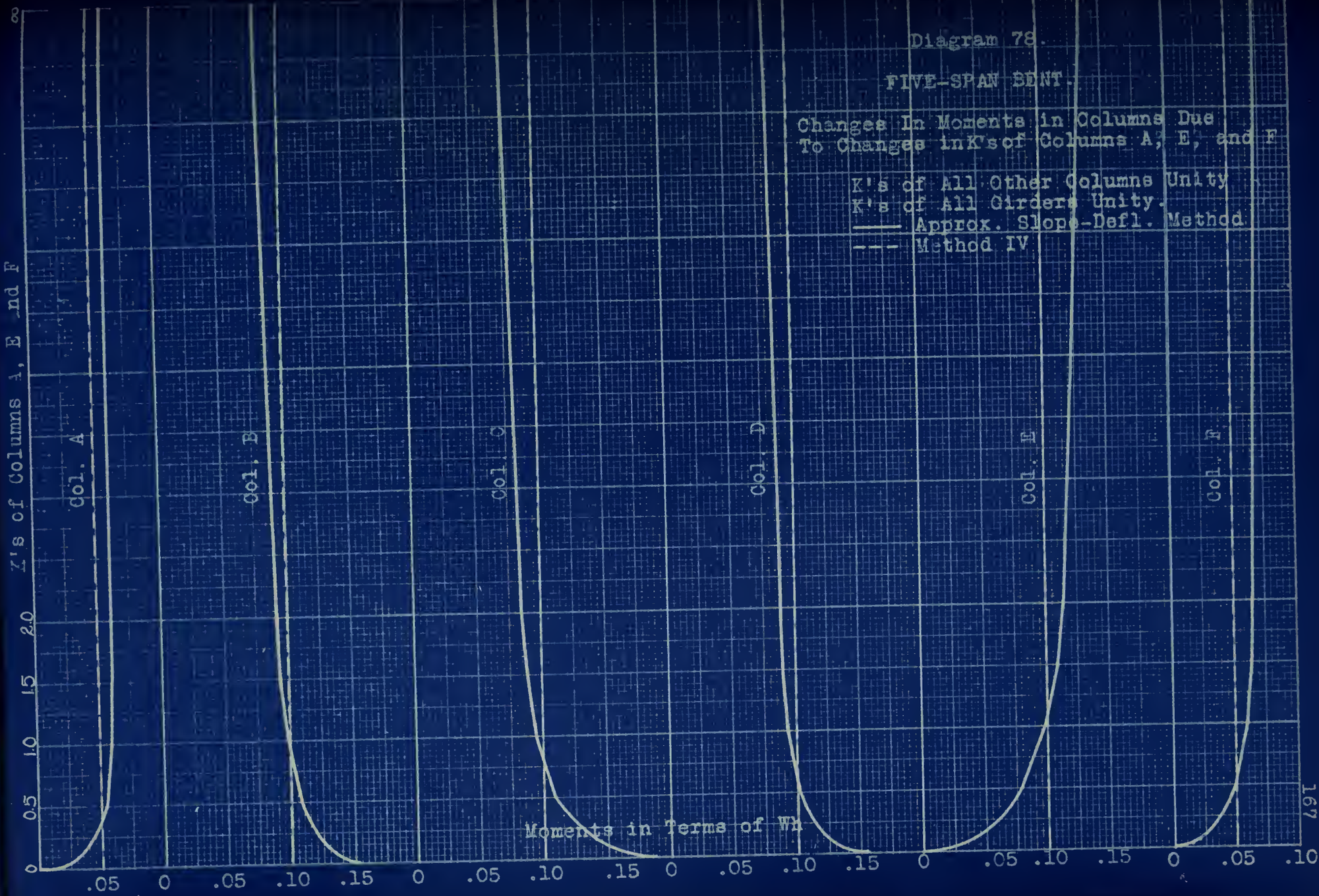


Diagram 78.
FIVE-SPAN BENT.
Changes In Moments in Columns Due
To Changes in K's of Columns A, E, and F
K's of All Other Columns Unity
K's of All Girders Unity.
Approx. Slope-Defl. Method
Method IV
K's of Columns A, E and F
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
.05
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10

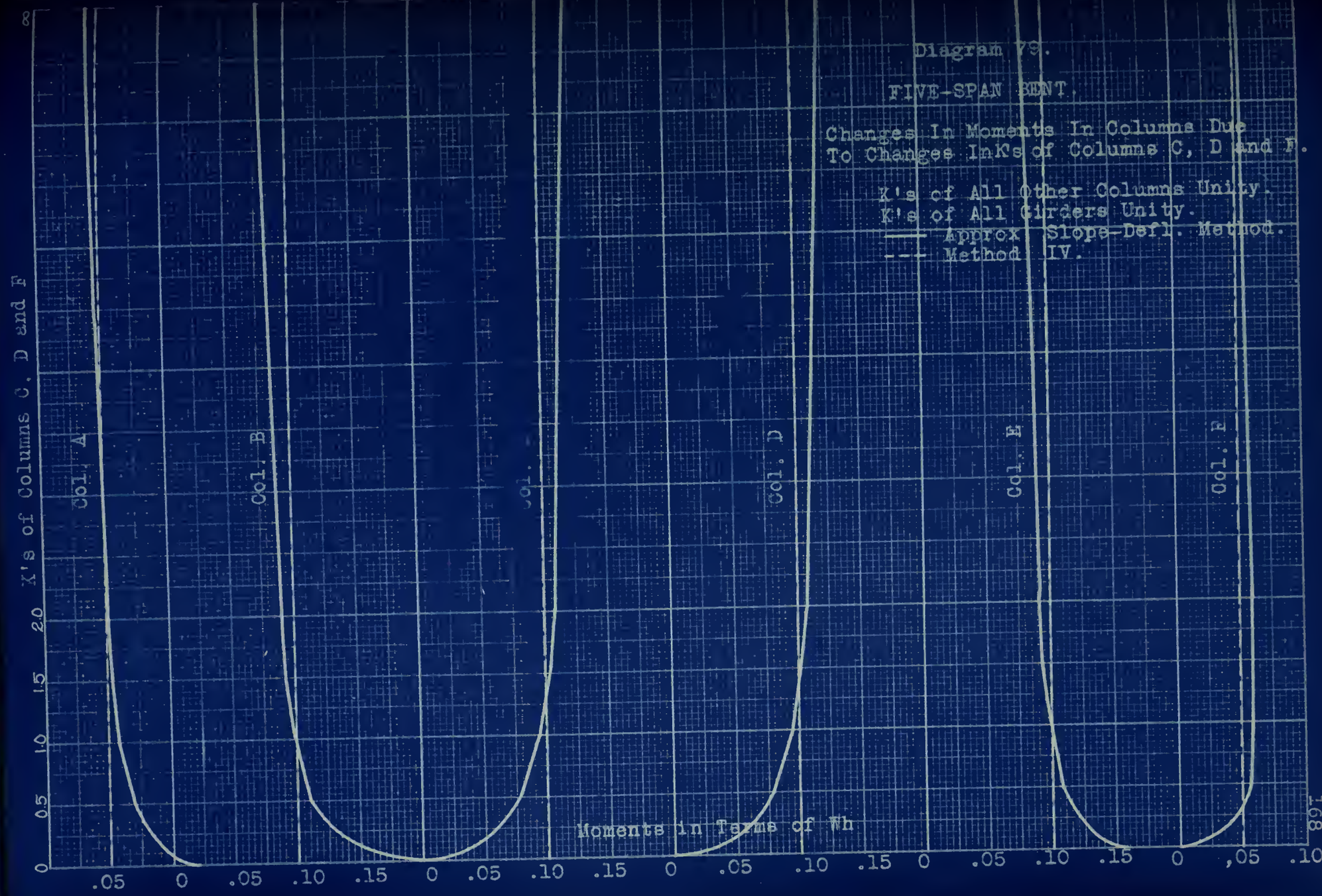


Diagram V9.
FIVE-SPAN BENT.
Changes In Moments In Columns Due To Changes InKs of Columns C, D and F.
K's of All Other Columns Unity.
K's of All Girders Unity.
—— Approx. Slope-Defl. Method.
--- Method IV.
Col. A
Col. B
Col. D
Col. E
Col. F
Moments In Terms of Wh
K's of Columns C, D and F
0
0.5
1.0
1.5
2.0
.05
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10

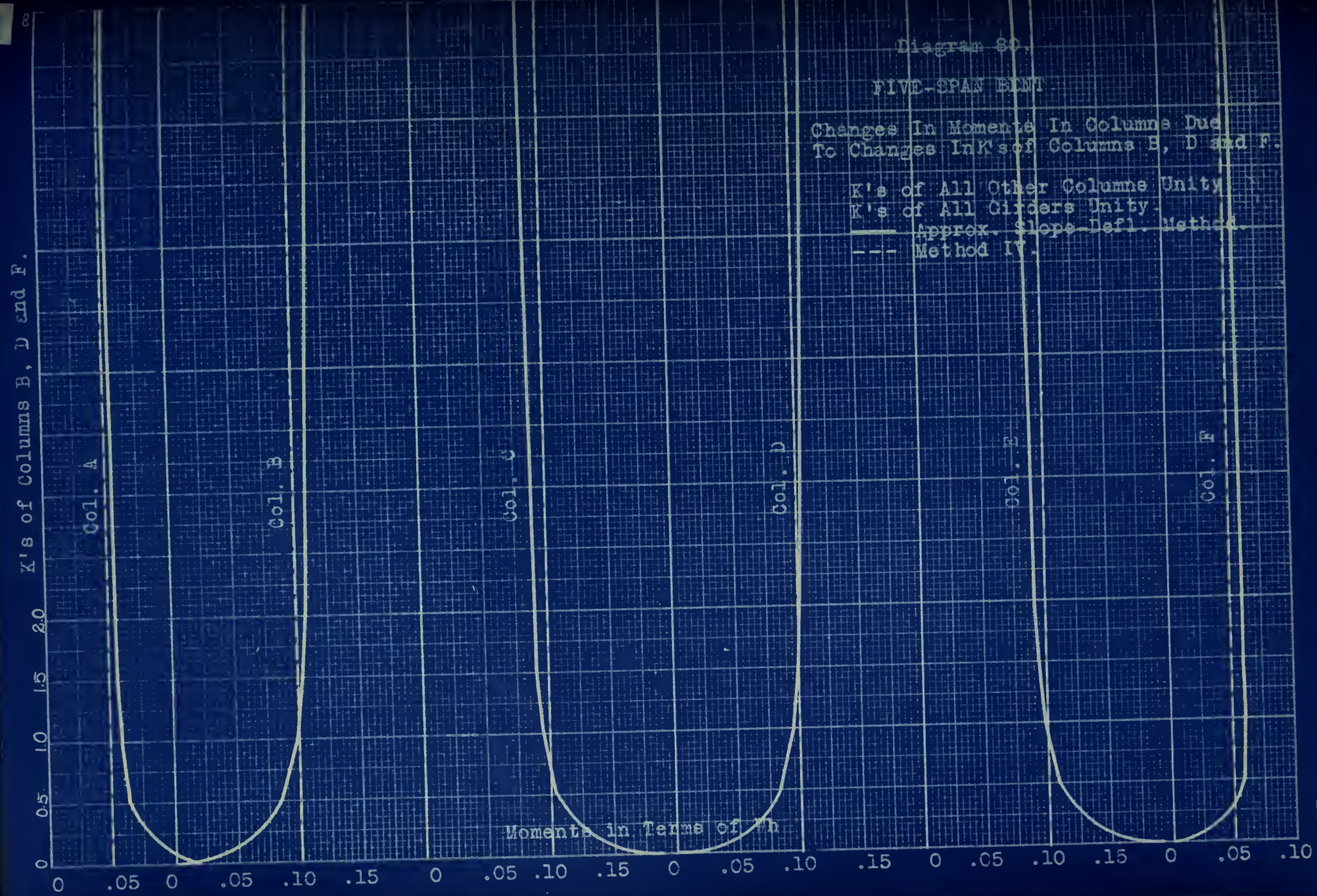


Diagram 8C.
FIVE-SPAN BENT
Changes In Moments In Columns Due To Changes In K's of Columns B, D and F.
K's of All Other Columns Unity
K's of All Girders Unity
—— Approx. Slope-Defl. Method.
--- Method IV.
K's of Columns B, D and F.
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Th
0
.05
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10
.15
0
.05
.10

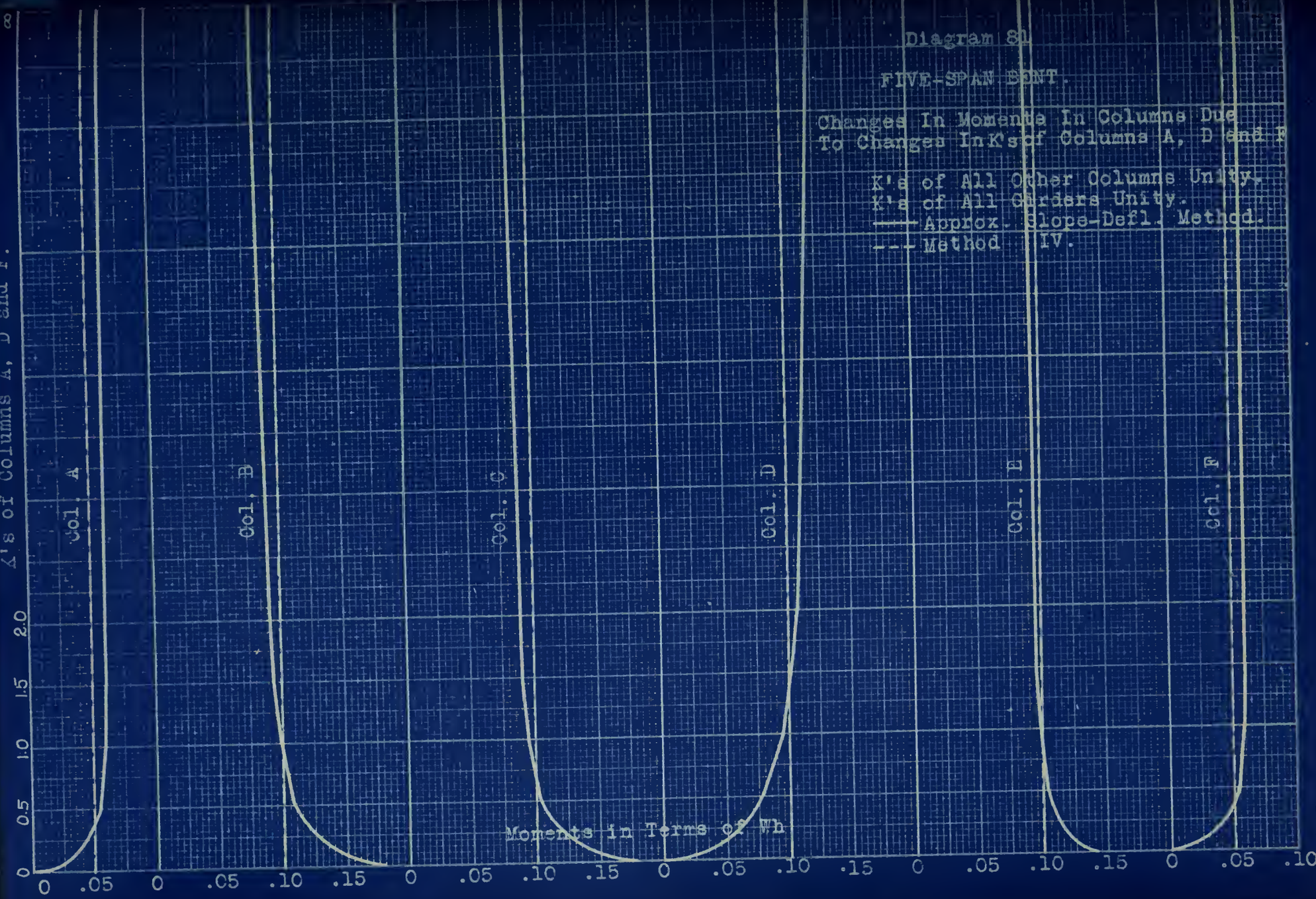


Diagram 81
FIVE-SPAN CONT.
Changes In Momente In Columns Due To Changes In K's of Columns A, D and F
K's of All Other Columns Unity.
K's of All Girders Unity.
Approx. Slope-Defl. Method.
Method IV.
K's of Columns A, D and F.
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
0
.05
.10
.15

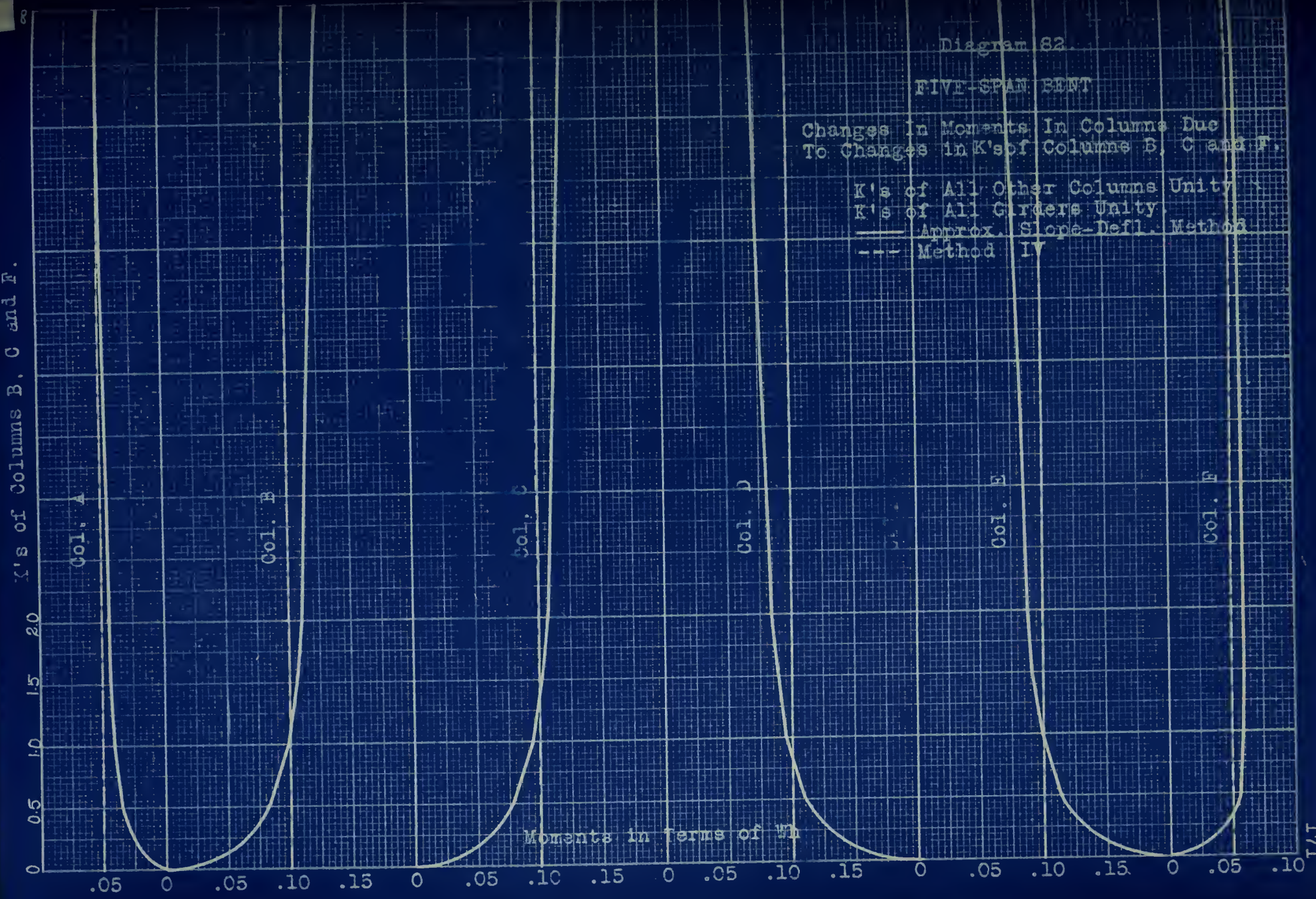


Diagram 82
FIVE-SPAN BENT
Changes In Moments In Columns Due To Changes in K's of Columns B, C and F.
K's of All Other Columns Unity
K's of All Girders Unity
Approx. Slope-Defl. Method
Method IV
K's of Columns B, C and F.
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
.05 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10

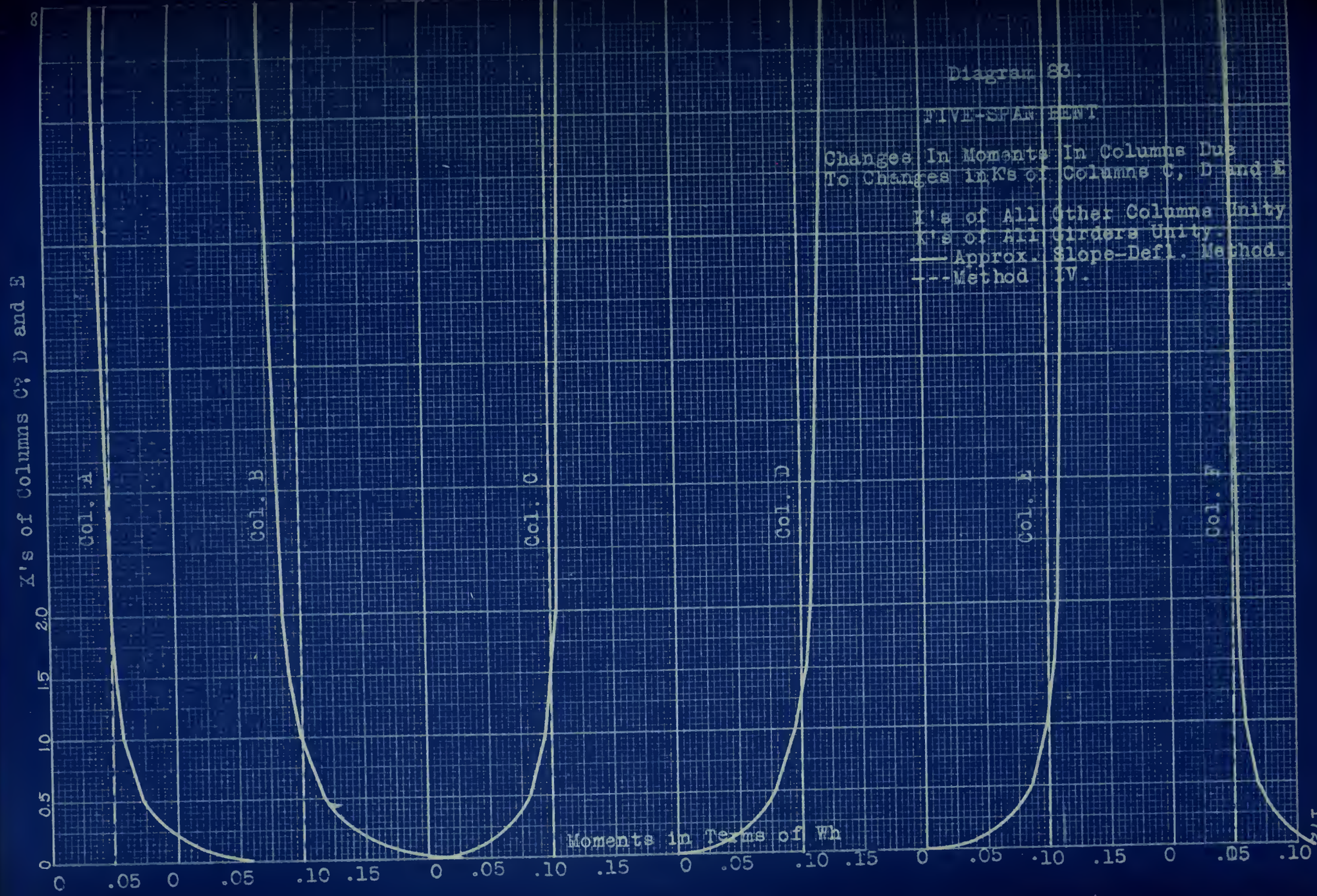


Diagram 83.
FIVE-SPAN BENT
Changes In Moments In Columns Due To Changes In K's of Columns C, D and E
K's of All Other Columns Unity
K's of All Girders Unity.
—— Approx. Slope-Defl. Method.
- - - Method IV.
K's of Columns C, D and E
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
0
0.5
1.0
1.5
2.0
0
.05
.10
.15

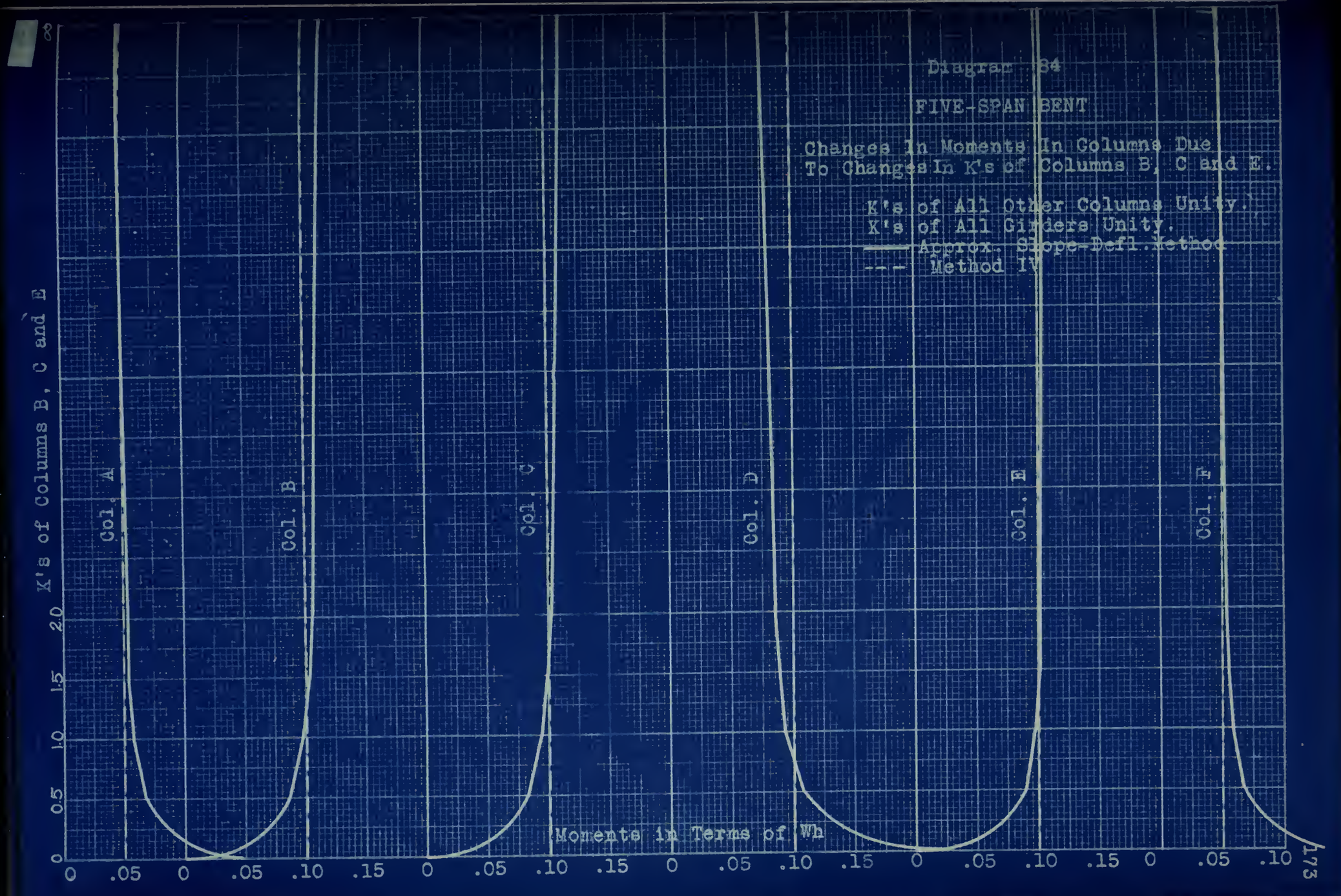


Diagram 84
FIVE-SPAN BENT
Changes In Moments In Columns Due To Changes In K's of Columns B, C and E.
K's of All Other Columns Unity.
K's of All Girders Unity.
Approx. Slope-Defl. Method
Method IV
K's of Columns B, C and E
0
0.5
1.0
1.5
2.0
Col. A
Col. B
Col. C
Col. D
Col. E
Col. F
Moments in Terms of Wh
0 .05 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10 .15 0 .05 .10

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Diagram 85

FIVE-SPAN BENT

Moments in Column A.

K of One Column Changes
K's of Other Columns Unity
K of Girder Unity
Numbers on Curves indicate magnitudes of K for member which varies.

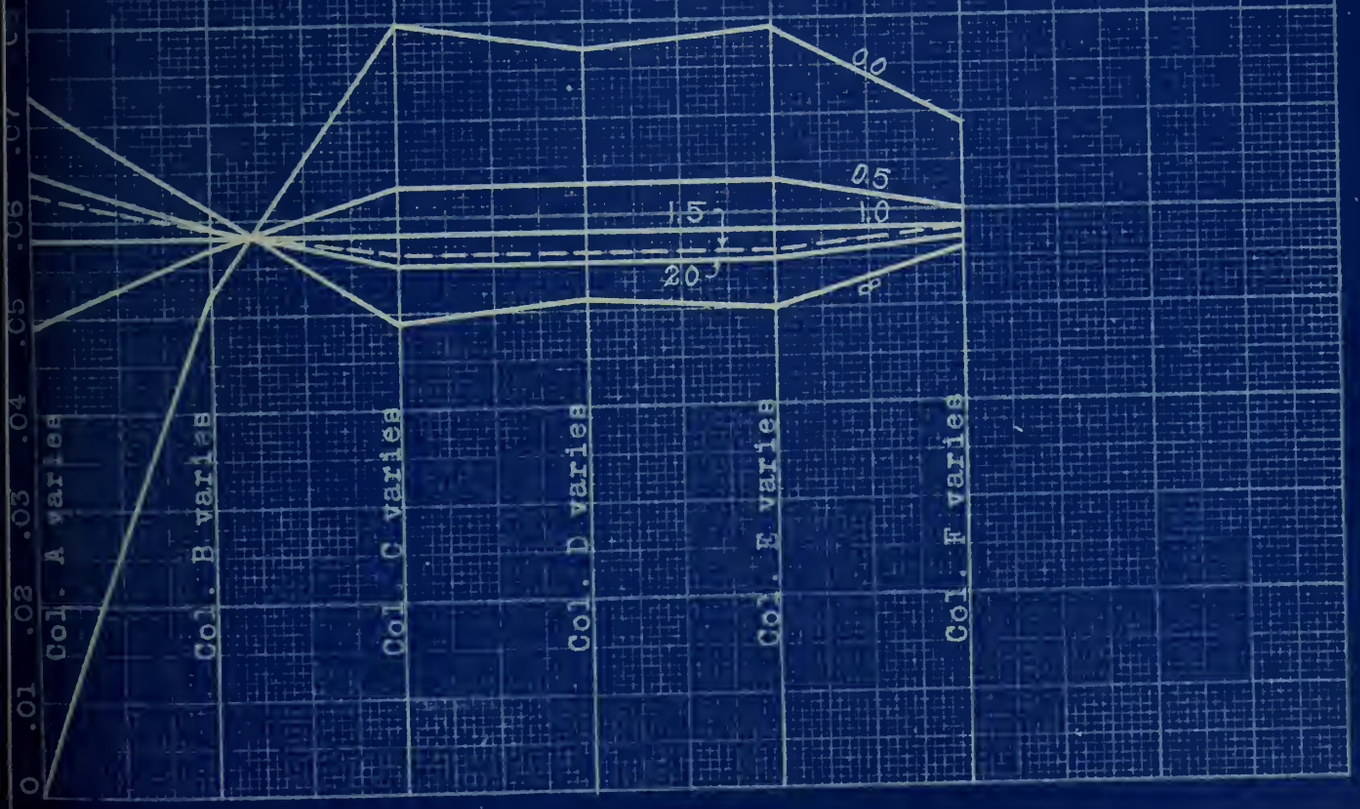

Diagram 86.

FIVE-SPAN BENT.

Moments In Column A.

K's of Column A and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 87.

FIVE-SPAN BENT

Moments in Column A.

K's of Column B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 88.

FIVE-SPAN BENT.

Moments in Column A.

K's of Column C and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary

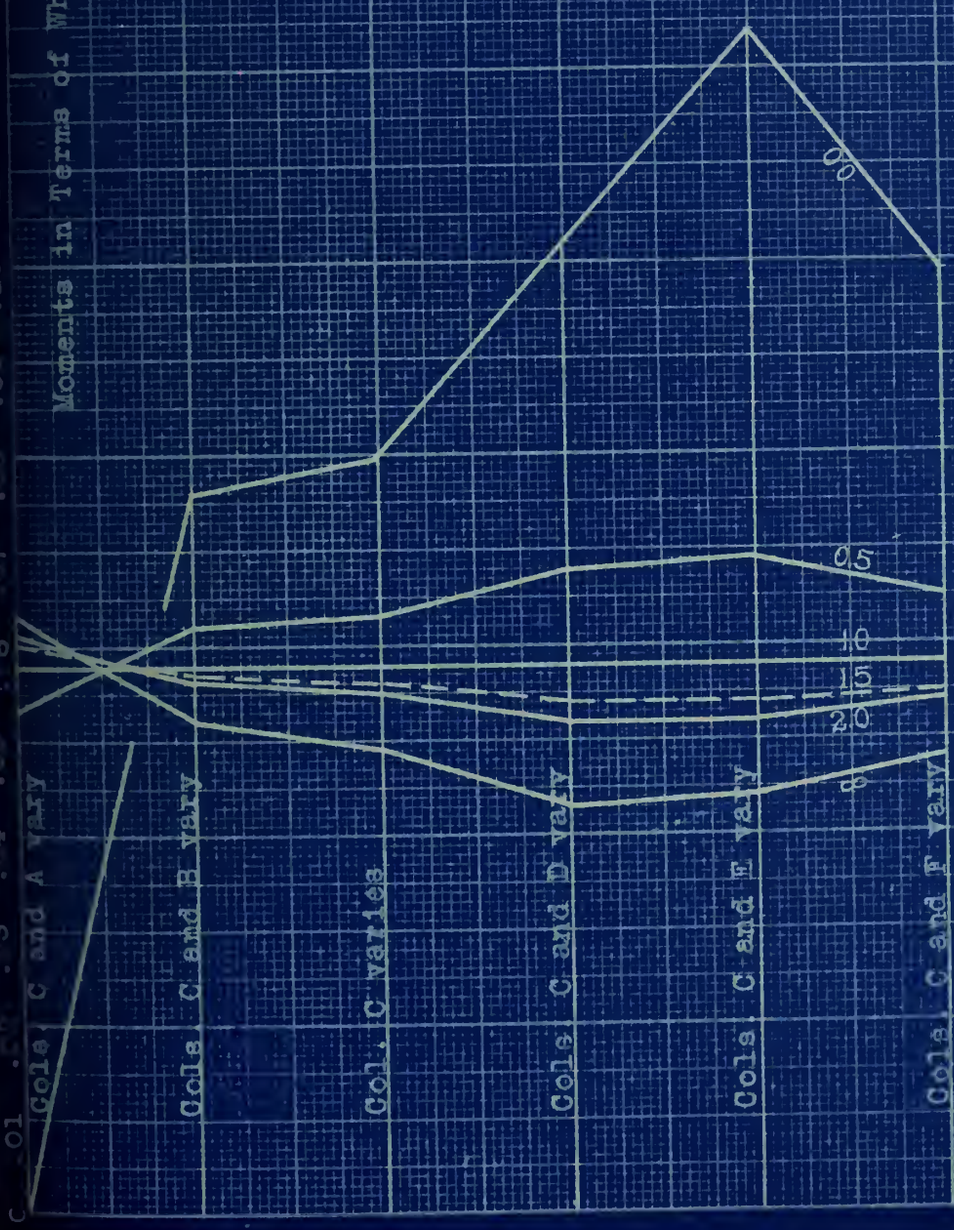

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Diagram 89.

FIVE-SPAN BENT.

Moments in Column A.

K's of Column D and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

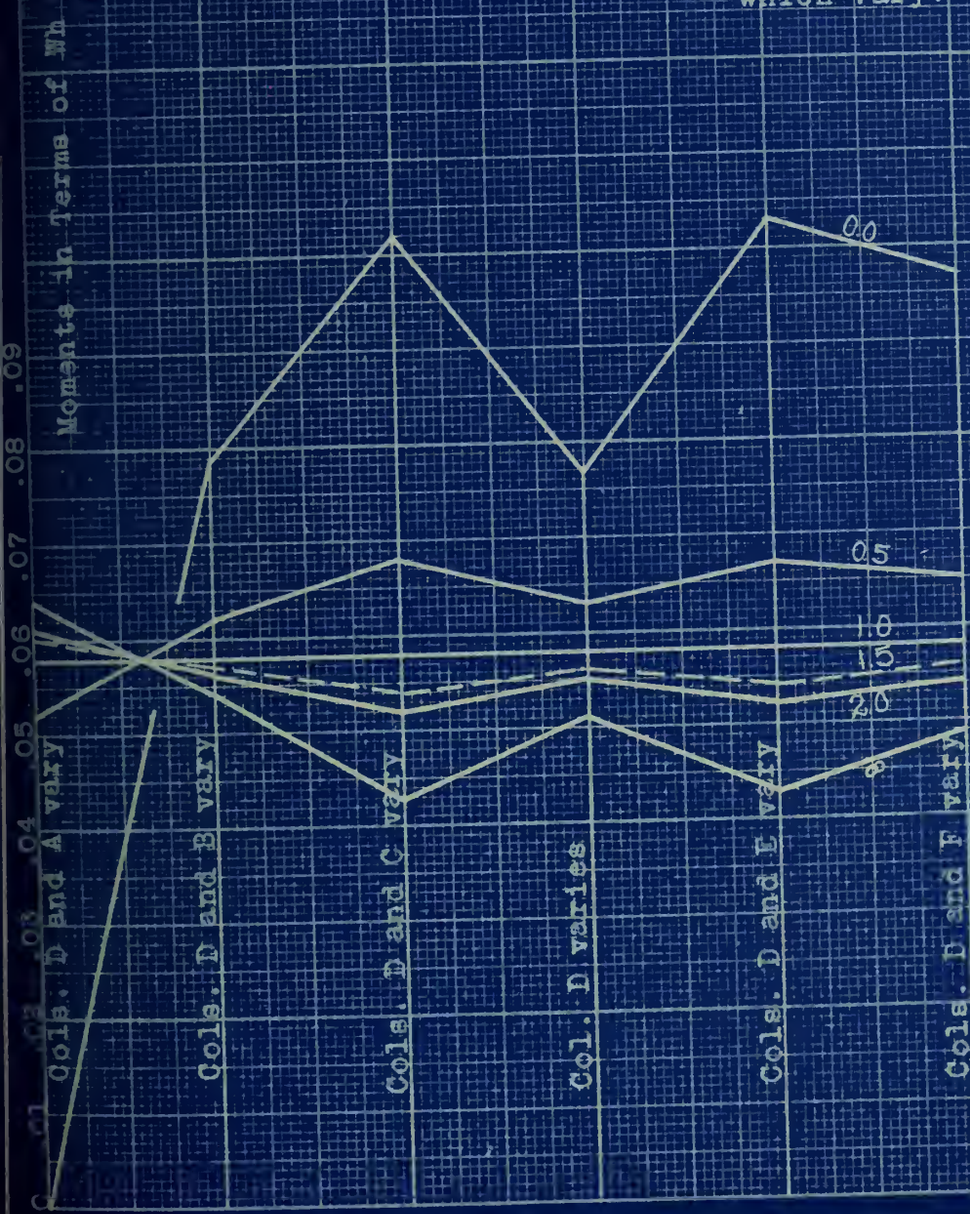

Diagram 90
FIVE-SPAN BENT.
Moments in Column A.
K's of Column E and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.
Moments in Terms of Wh.
0.01
.02
.03
Cols. E and A vary
Cols. E and B vary
Cols. E and C vary
Cols. E and D vary
Col. E varies
Cols. E and F vary
0.0
0.5
1.0
1.5
2.0
∞

Diagram 91.

FIVE-SPAN BENT.

Moments in Column A.

K's of Column F and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

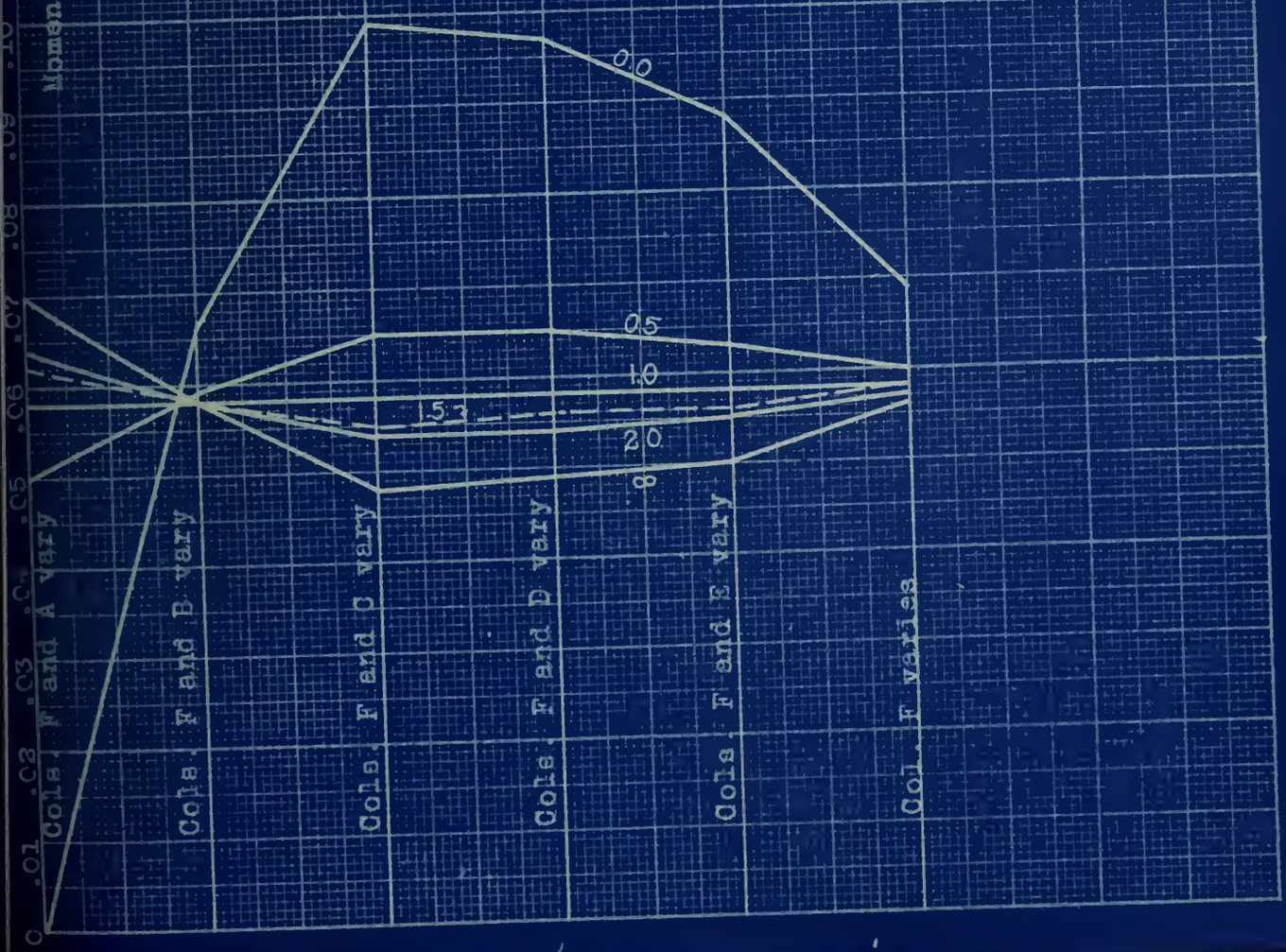

Diagram 92.

FIVE-SPAN BENT

Moments in Column A.

K's of Column A, B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary

Diagram 93.

FIVE SPAN BENT

Moments In Colum A.

K's of Colums A, C, and One Other Colum Change Simultaneously.
K's of All Other Colums Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 94.

FIVE-SPAN BENT

Moments in Columns A.

K's of Columns A, D, and One Other Colum Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 95

FIVE-SPAN BENT

Moments In Colum A.

K's of Columns A, E and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 96.

FIVE-SPAN BENT

Moments In Column A.

K's of Columns A, F, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary

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Diagram 97.

FIVE-SPAN BENT

Moments in Column A.

K's of Columns B, C, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 98.

FIVE-SPAN BENT

Moments in Column A.

K's of Columns B, D and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 89.

FIVE-SPAN BENT.

Moments In Column A.

K's of Columns B, E and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 100.

FIVE-SPAN BENT

Moments In Column A.

K's of Columns B, F and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 101.

FIVE-SPAN BENT

Moments In Column A.

K's of Columns C, D, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 102.
FIVE-SPAN BENT
Moments In Column A.
K's of Columns C, E and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.
Moments in Terms of Wh
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
.15
.16
.17
.18
Cols. C, E and A vary
Cols. C, E and B vary
Cols. C and E vary
Cols. C, E and D vary
Cols. C and E vary
Cols. C, E and F vary
0.0
0.5
1.0
1.5
2.0
8

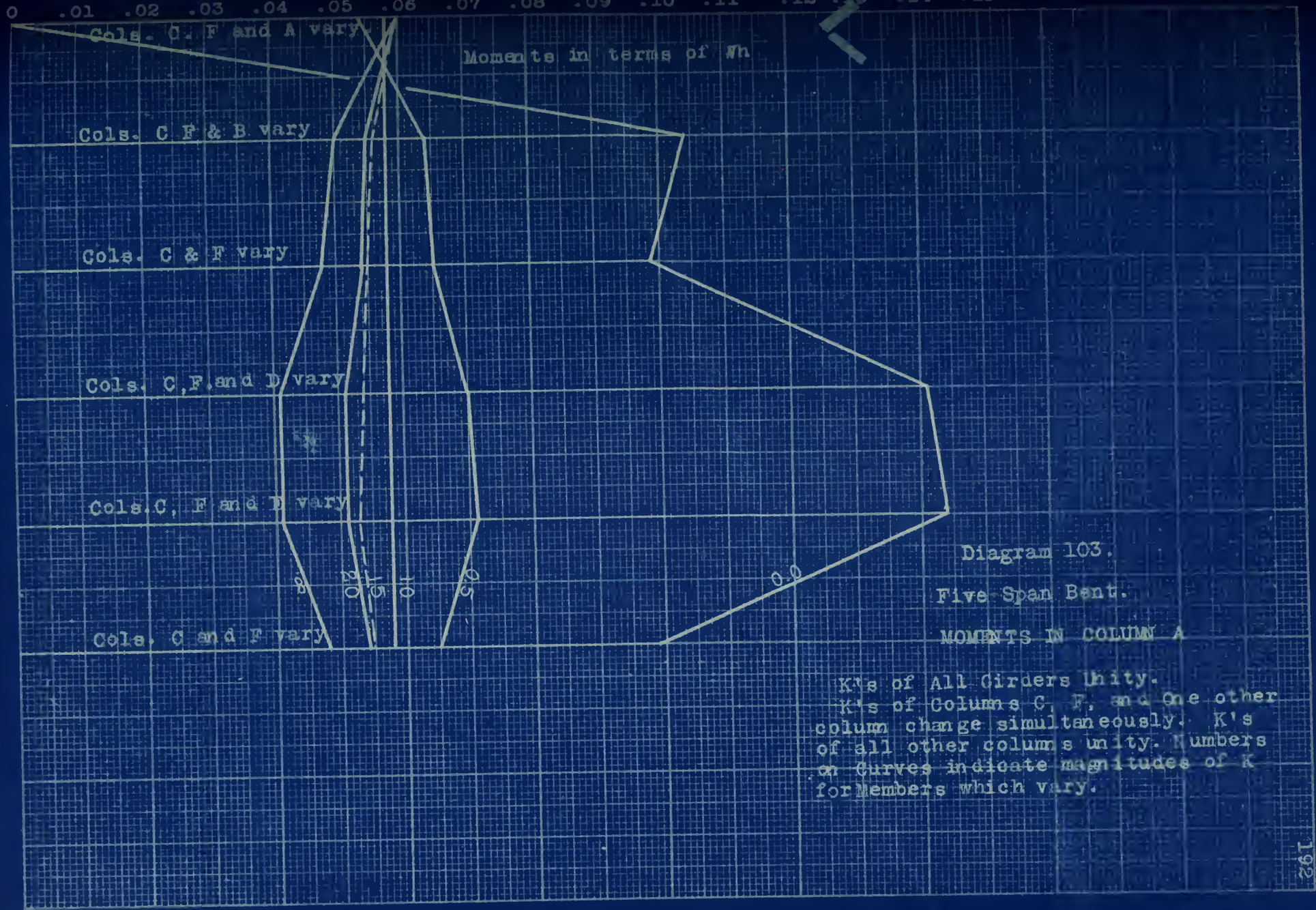

Diagram 103.

Five Span Bent.

MOMENTS IN COLUMN A

K's of All Girders Unity.
K's of Columns C, F, and One other column change simultaneously. K's of all other columns unity. Numbers on Curves indicate magnitudes of K for Members which vary.

Diagram 104.

FIVE-SPAN BENT.

Moments In Column A.

K's of Columns D, E and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K for members which vary.

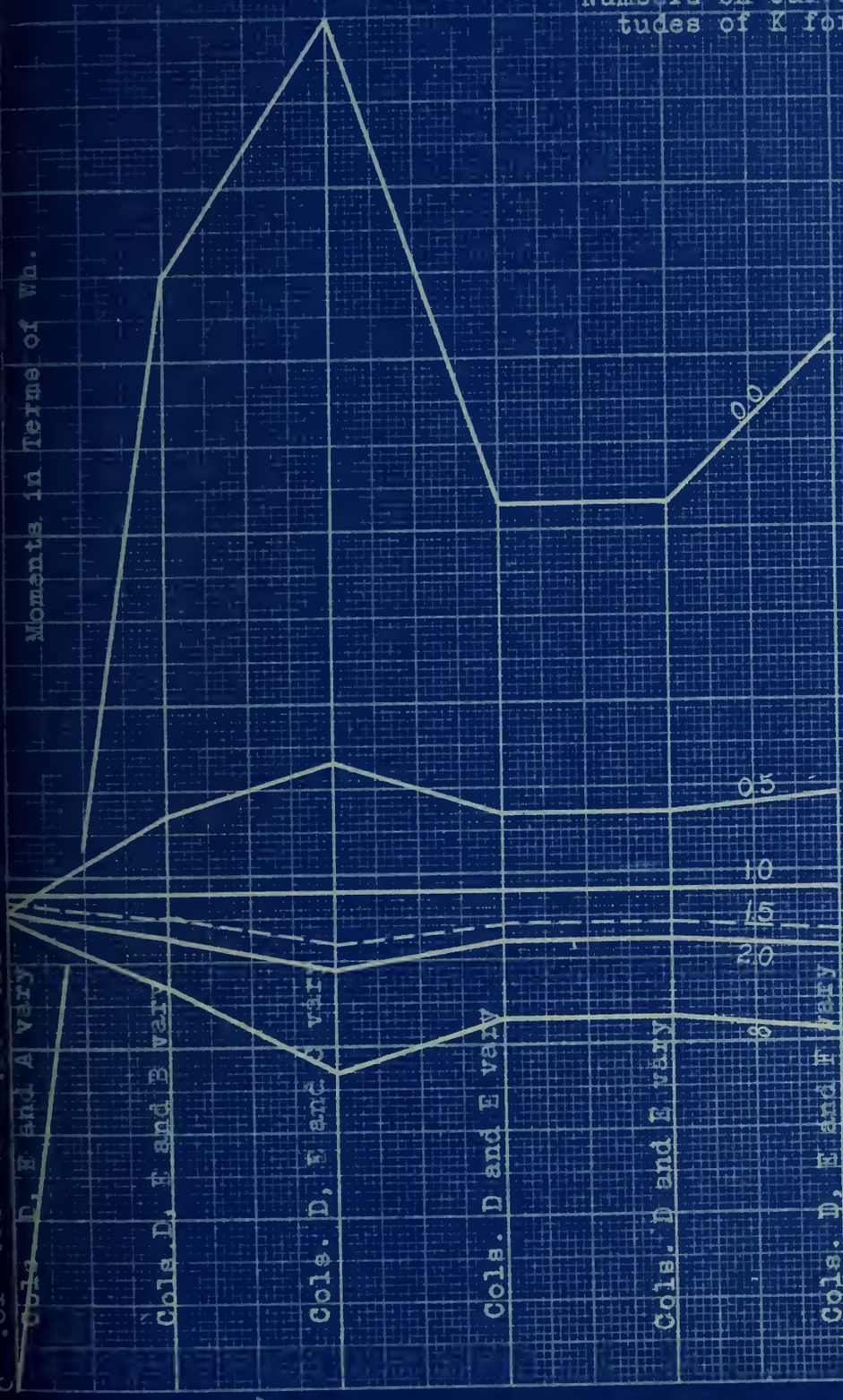

Diagram 105.

FIVE-SPAN BENT.

Moments In Column A.

K's of Columns D, F, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K for members which vary.

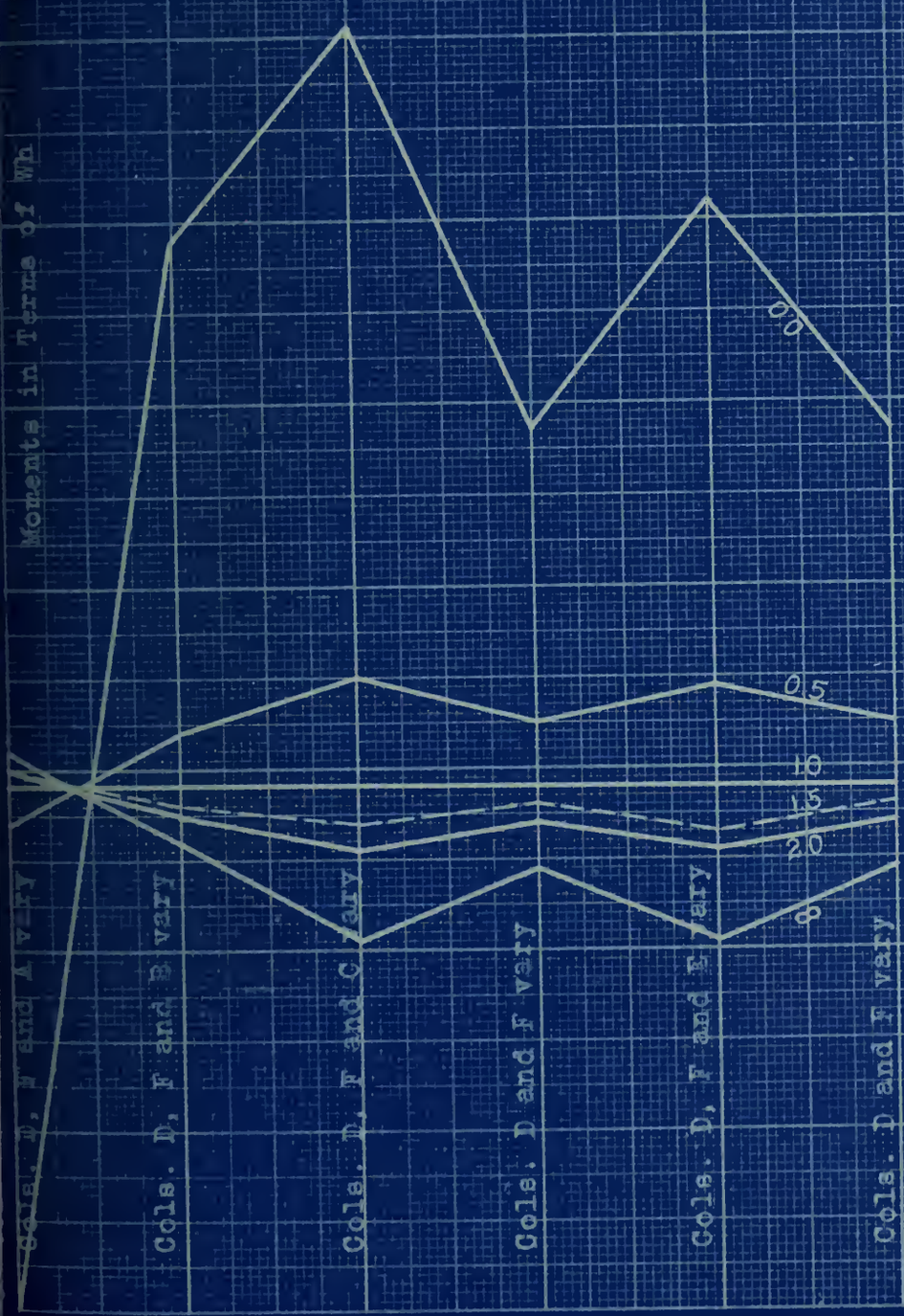

Diagram 106

FIVE-SPAN BENT

Moments In Column A.

K's of Columns E, F and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K for members which vary

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Diagram 107

FIVE-SPAN BENT

Moments in Column B

K of One Column Changes
K's of Other Columns
unity.
K of Girder Unity.
Numbers on Curves
indicate magnitudes
of K for member
which varies.

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Diagram 108

FIVE-SPAN BENT

Moments In Column B.

K's of Column A and One Other Column Change Simultaneously.
K's of All Other Columns Unity
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 109.

FIVE-SPAN BENT.

Moments in Column B.

K's of Column B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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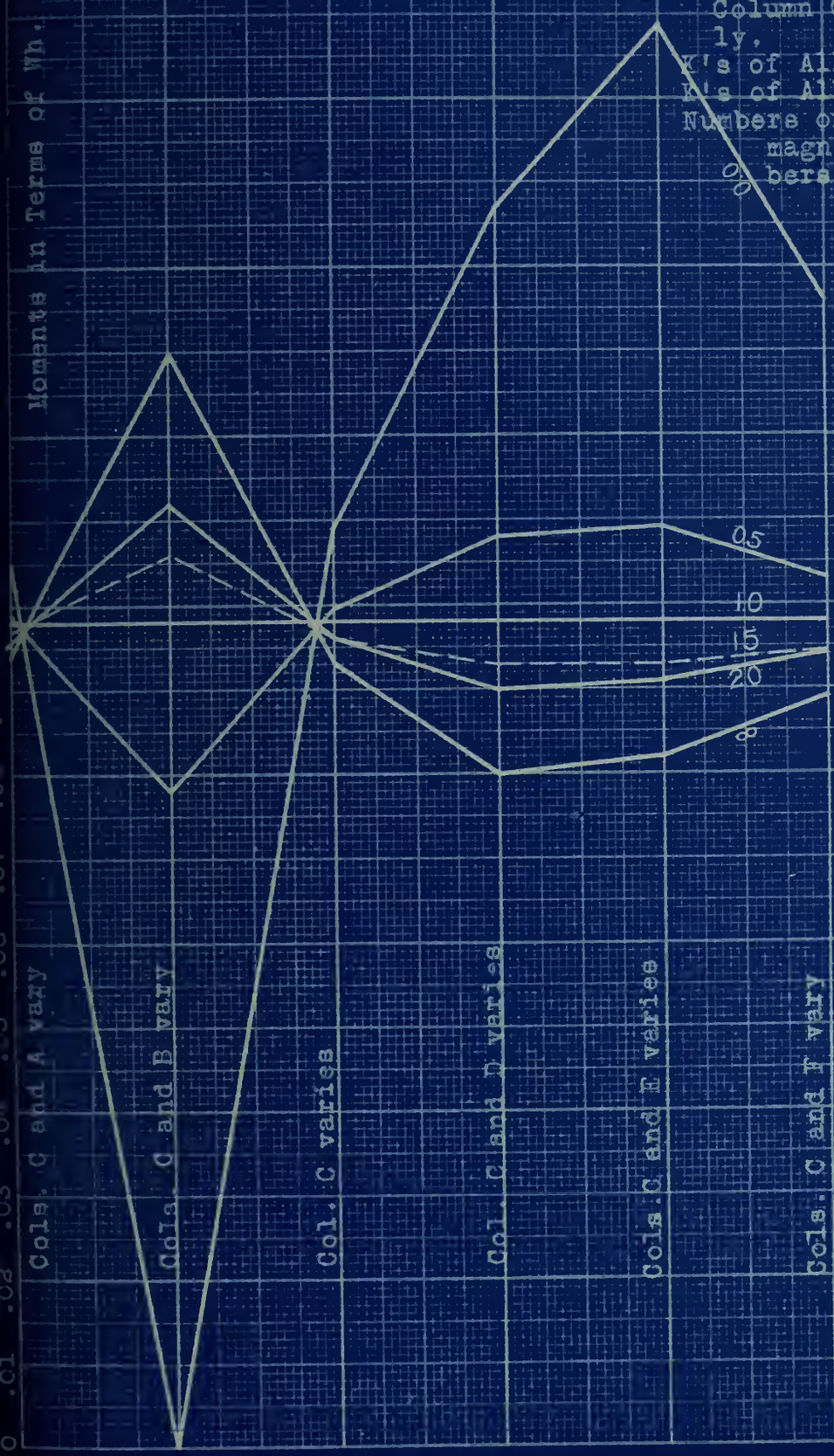
Diagram 110.

FIVE-SPAN BENT.

Moments in Column B.

K's of Column C and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.



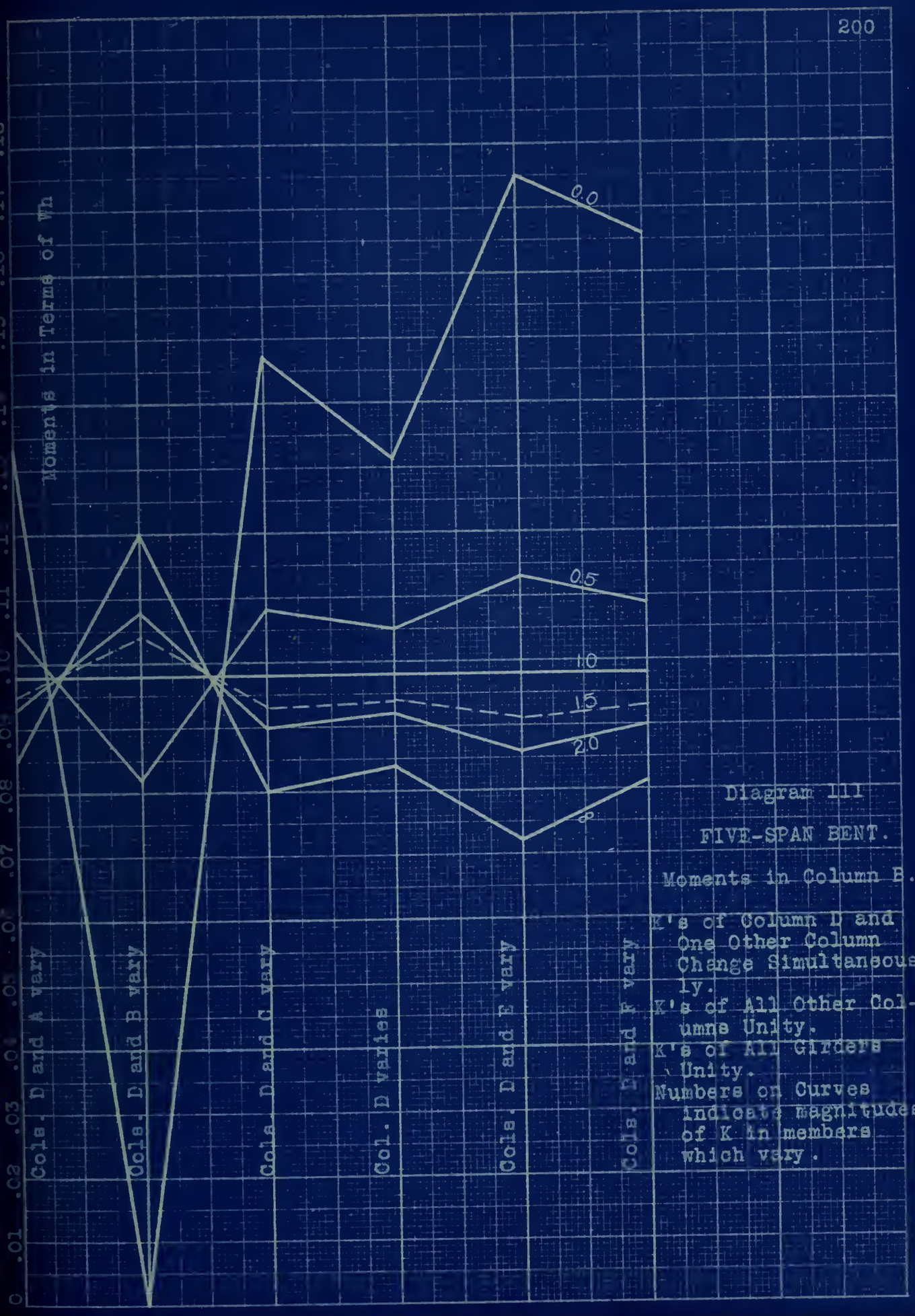


Diagram III

FIVE-SPAN BENT.

Moments in Column B.

K's of Column D and One Other Column Change Simultaneously.

K's of All Other Columns Unity.

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 112.

FIVE-SPAN BENT

Moments in Column B.

K's of Column E and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

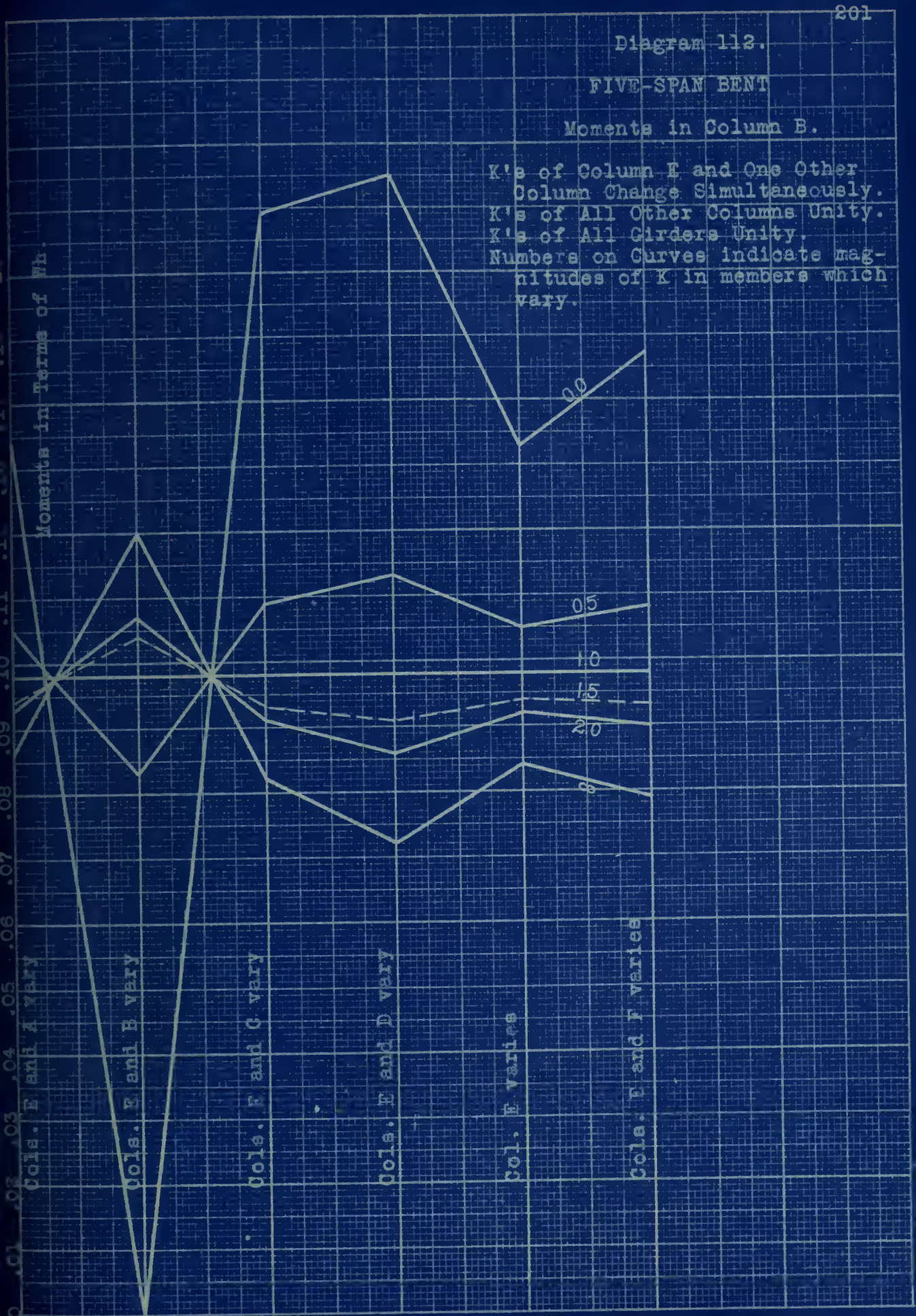

Diagram 113.

FIVE-SPAN BENT

Moments In Column B.

K's of Column F and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

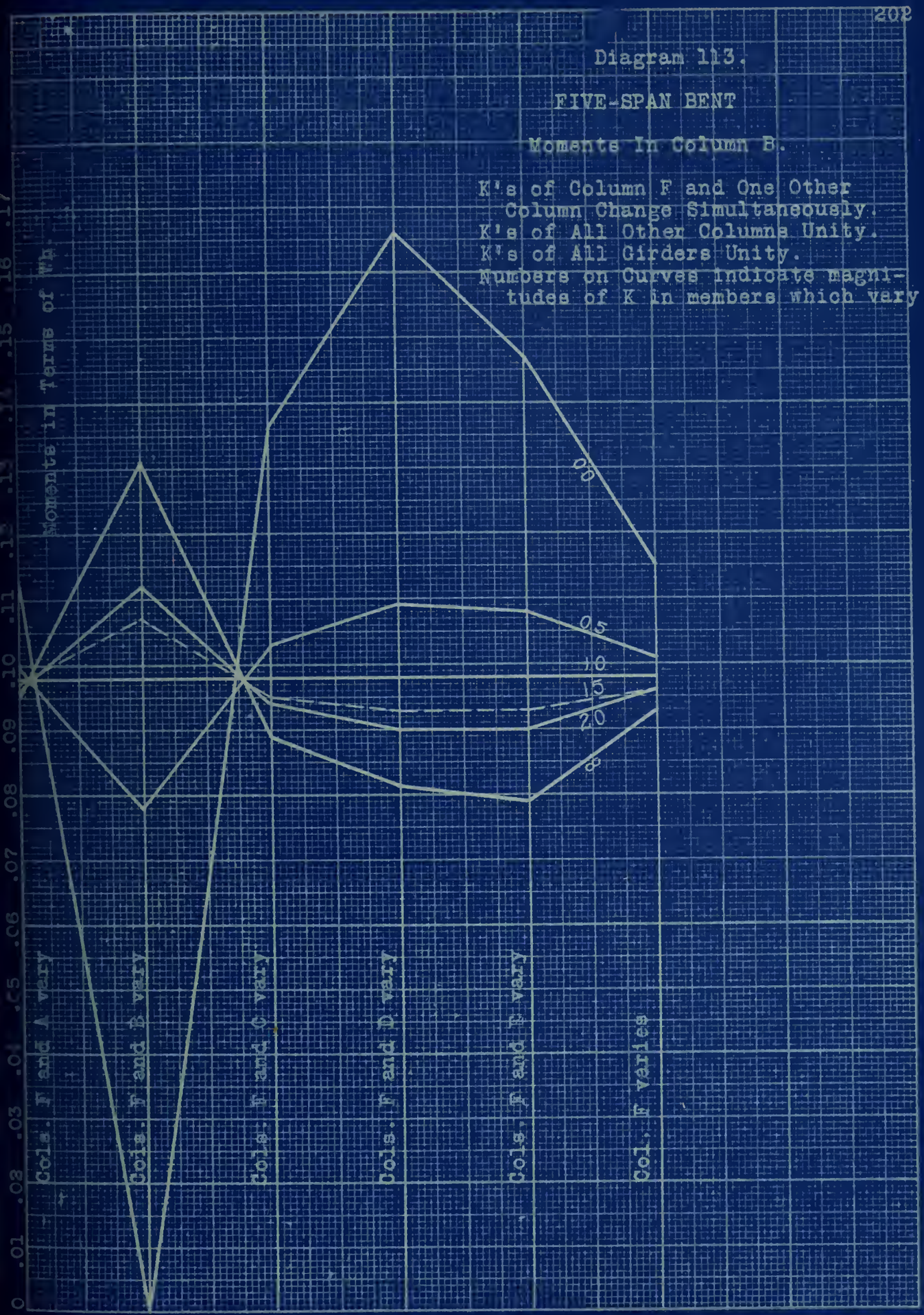

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Diagram 11a.

FIVE-SPAN BENT

Moments In Column B.

K's of Columns A, B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 115.

FIVE-SPAN BENT

Moments in Column B.

K's of Columns A, C, and One Other Column Change Simultaneously.

K's of All Other Columns Unity.

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 116.

FIVE-SPAN BENT.

Moments In Column B.

K's of Columns A, D, and One Other Column Change Simultaneously.

K's of All Other Columns Unity.

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 117.

FIVE-SPAN BENT.

Moments In Column B.

K's of Columns A, E, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 118.

FIVE-SPAN BENT.

Moments in Column B.

K's of Columns A, F and One Other Column Change Simultaneously.

K's of All Other Columns Unity.

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary.

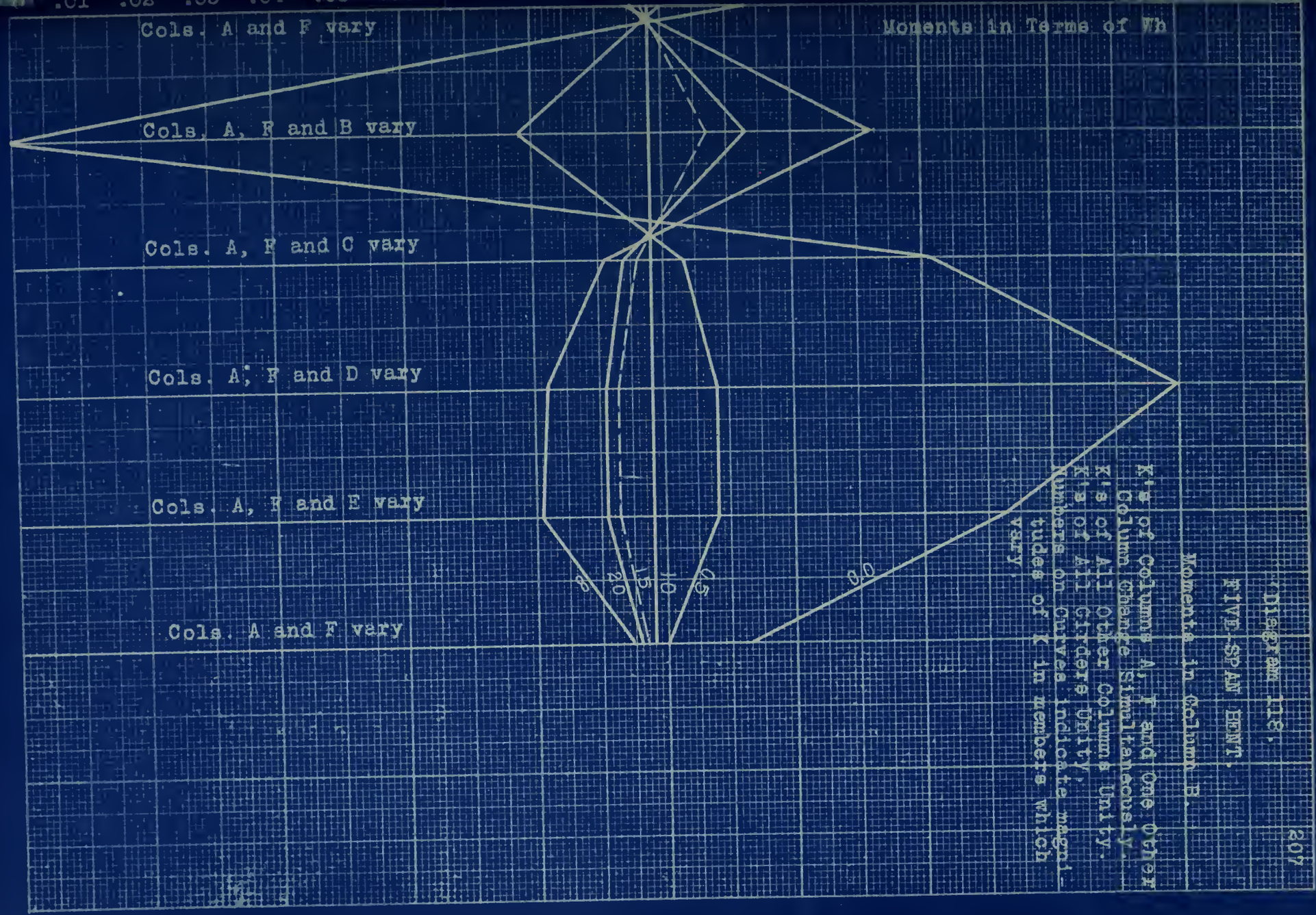

Diagram 119.

FIVE-SPAN BENT.

Moments In Column B.

K's of Columns B, C and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 120.

FIVE-SPAN BENT.

Moments In Column B.

K's of Columns B, D and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 121.

FIVE-SPAN BENT

Moments In Column B.

K's of Columns B, E and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 122.

FIVE-SPAN BENT.

Moments in Column B.

K's of Columns B, F, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves Indicate magnitudes of K in members which vary.

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Diagram 123.

FIVE-SPAN BENT

Moments in Column B.

K's of Columns C, D, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 124.

FIVE SPAN BENT

Moments In Column B

I's of Columns C, E, and One Other Column Change Simultaneously.
I's of All Other Columns Unity.
I's of All Girders Unity.
Numbers on Curves indicate magnitudes of I in members which vary.

Diagram 126.

FIVE-SPAN BENT.

Moments in Column B

K's of Columns C, F, and One Other Column Change Simultaneously.

K's of All Other Columns Unity.

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 126.

FIVE-SPAN BENT

Moments in Column B

K's of Columns D, E, and One Other Column Change Simultaneously.

K's of All Other Columns Unity.

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 127.

FIVE-SPAN BENT.

Moments in Column B.

K's of Columns D, F, and One Other Column Change Simultaneously.

K's of All Other Columns Unity.

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 128.

FIVE-SPAN BENT.

Moments in Column B

K's of Columns E, F and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 129.
FIVE SPAN BENT
Moments in Columns C.
K of One Column Changes.
K's of Other Columns Unity.
K of Girder Unity.
Numbers on Curves indicate magnitudes of K for member which varies.
Moments in Terms of Wh
0 .01 .02 .03 .04 .05 .06 .07 .08 .09 .10 .11 .12 .13
Col. A varies
Col. B varies
Col. C varies
Col. D varies
Col. E varies
Col. F varies
∞
0.5
1.0
1.5
2.0
∞

Diagram 130.

FIVE-SPAN BENT

Moments in Column C

K's of Column A and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 131.

FIVE-SPAN BENT

Moments in Column C

K's of Column B and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 132.

FIVE-SPAN BENT.

Moments in Column C

K's of Column C and One Other Column Change Simultaneously
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 133.

FIVE-SPAN BENT.

Moments in Column C

K's of Column D and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

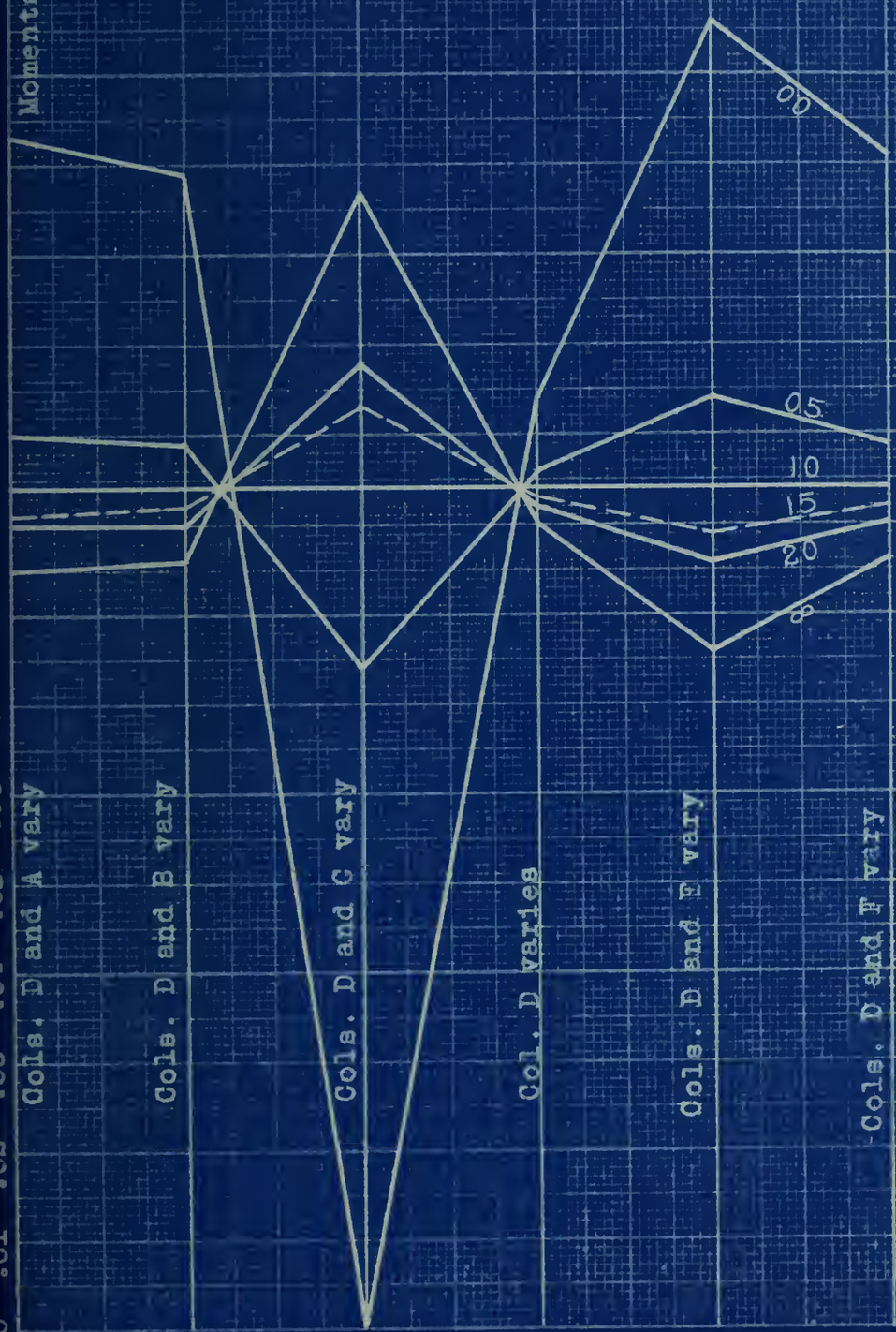

Diagram 134.

FIVE-SPAN BENT.

Moments in Column C.

K's of Column E and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

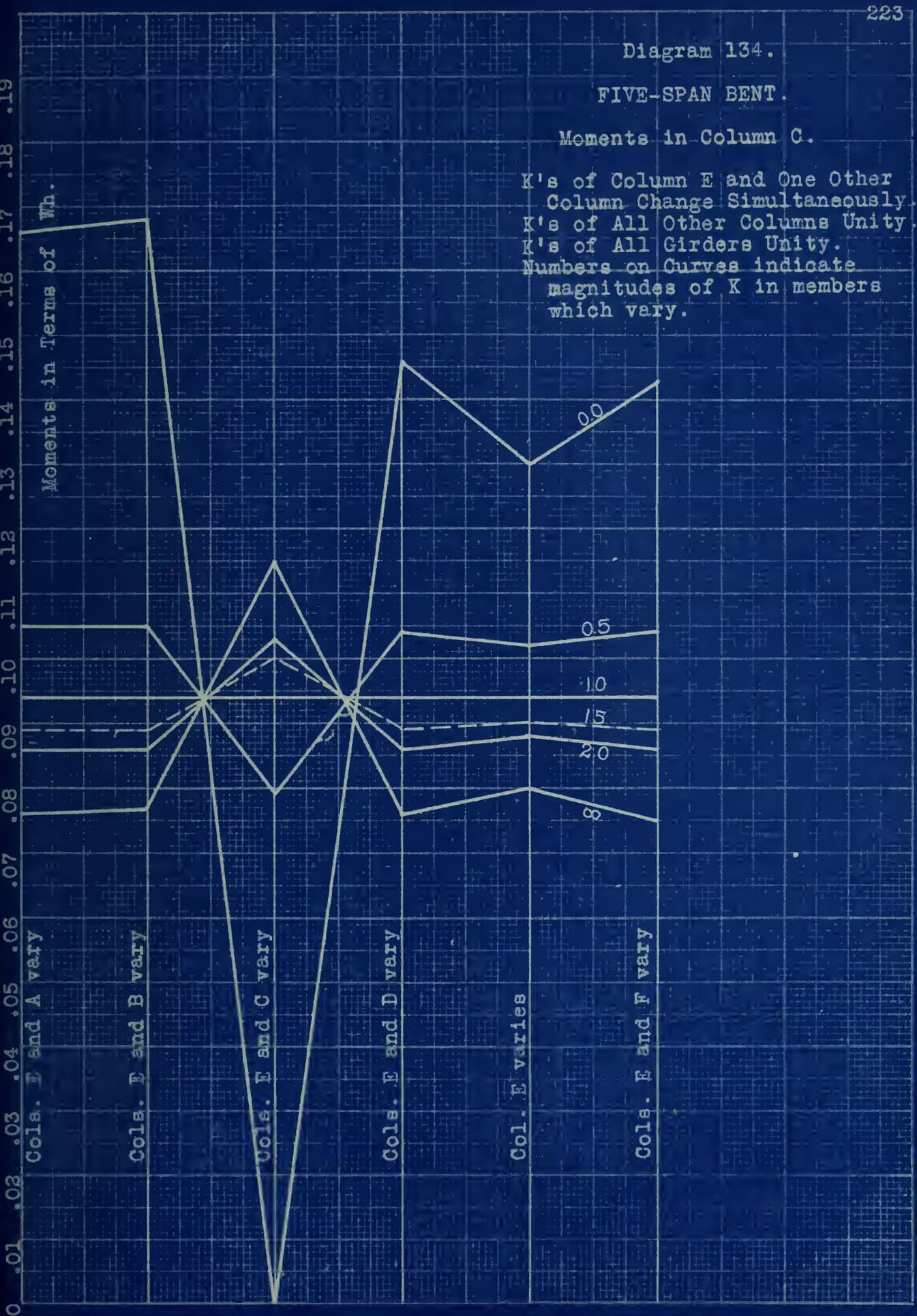

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Diagram 135.

FIVE-SPAN BENT.

Moments in Column C.

K's of Column F and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary

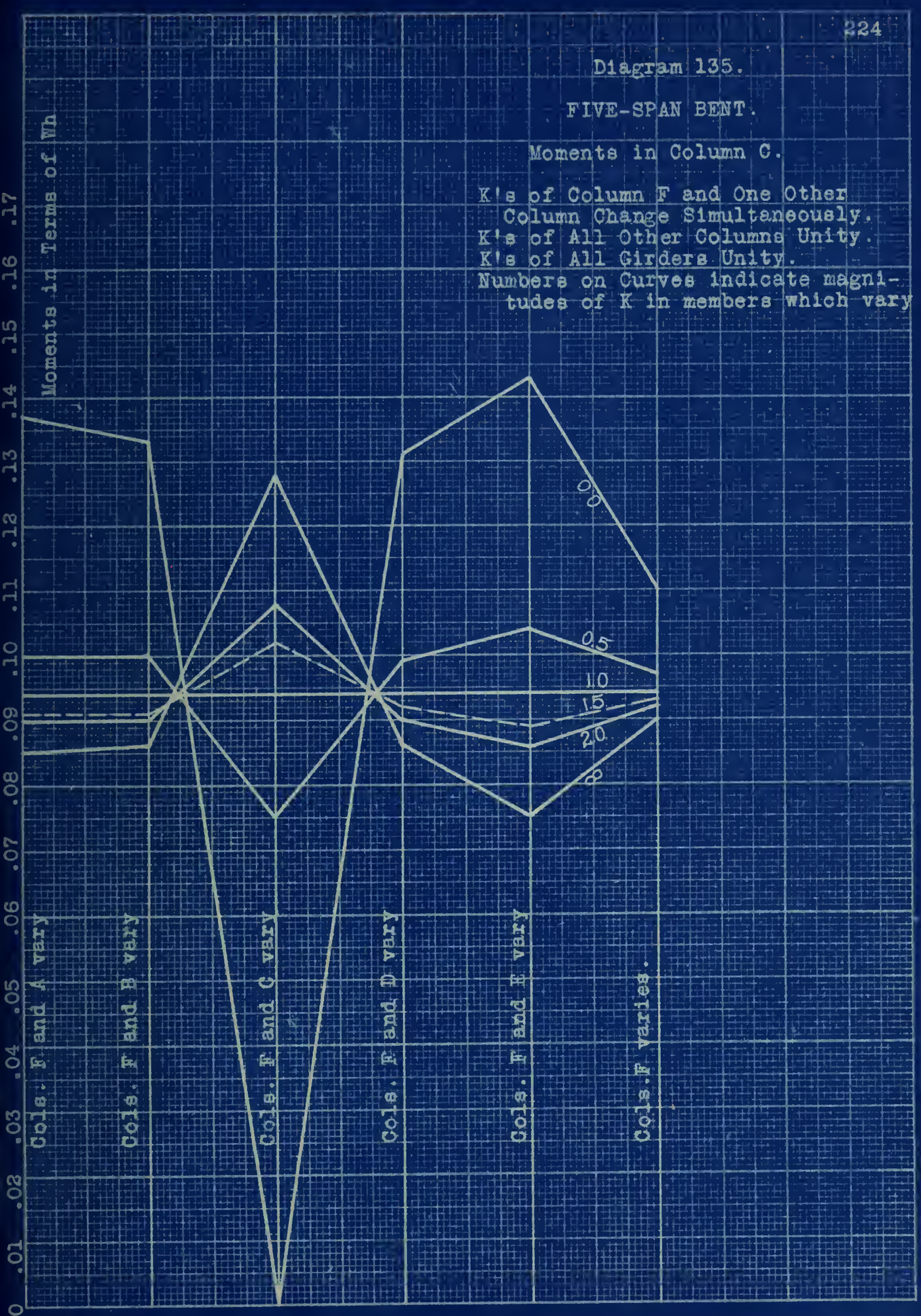

Diagram 136.

FIVE-SPAN BENT.

Moments In Column C

K's of Columns A, B, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 137.

FIVE-SPAN BENT.

Moments in Column C.

K's of Columns A, C, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

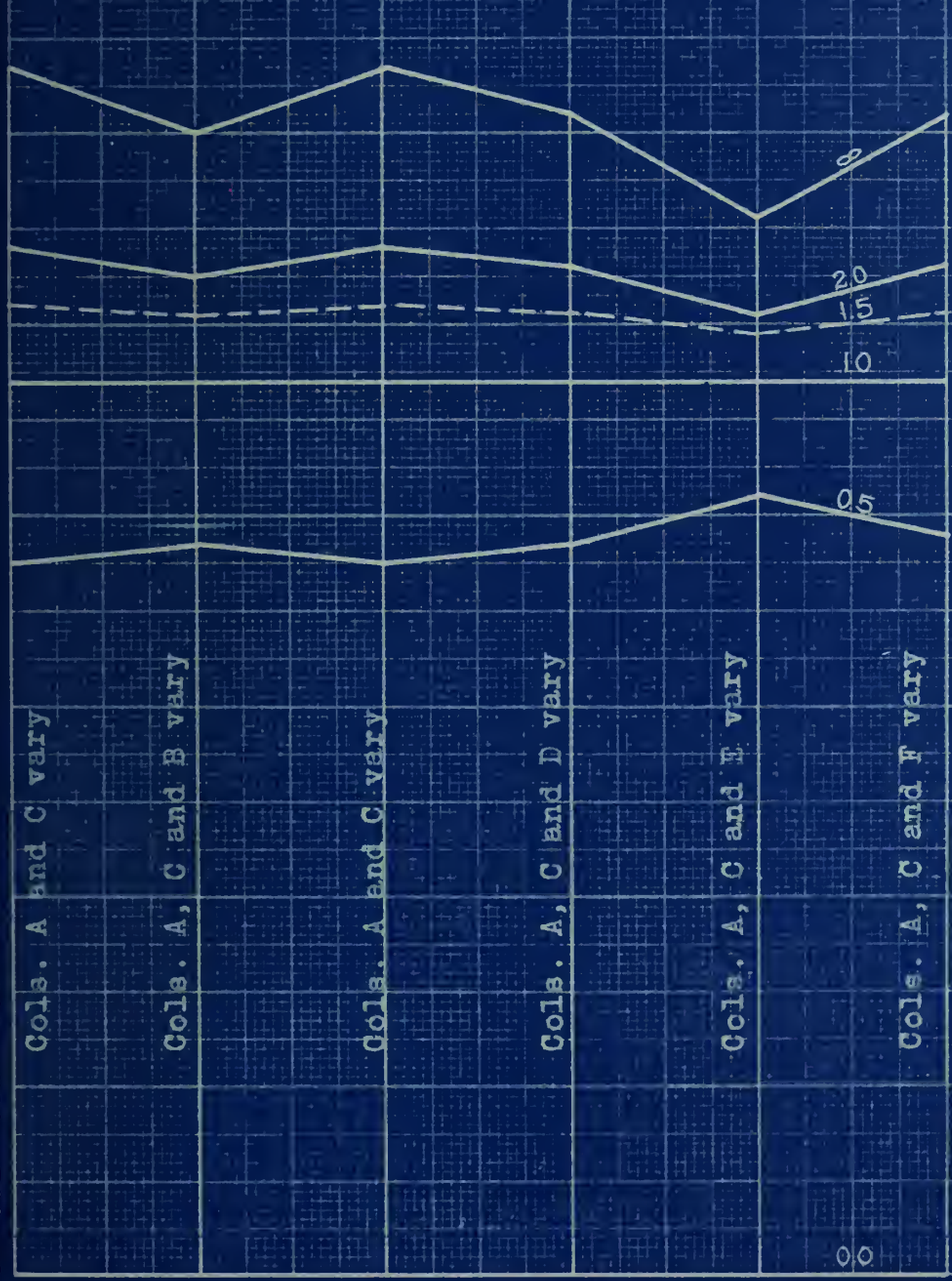

Diagram 138.

FIVE-SPAN BENT.

Moments In Column C

K's of Columns A, D, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 139.

FIVE-SPAN BENT.

Moments In Column C

K's of Columns A, E, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 140.

FIVE-SPAN BENT.

Moments in Column C.

K's of Columns A, F, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 141.
FIVE-SPAN BENT.
Moments in Column C.
K's of Columns B, C and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary
Moments in Terms of Wh
0
.01
.02
.03
.04
.05
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.07
.08
.09
.10
.11
.12
.13
.14
.15
Cols. B, C and A vary
Cols. B and C vary
Cols. B and C vary
Cols. B, C and D vary
Cols. B, C and E vary
Cols. B, C and F vary
∞
2.0
1.5
1.0
0.5
0.0

Diagram 142.

FIVE-SPAN BENT.

Moments In Column C

K's of Columns B, D, and One Other Column Change Simultaneously.

K's of All Other Columns Unity.

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary.

Diagram 143.

FIVE-SPAN BENT.

Moments In Column C

K's of Columns B, E, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 144.

FIVE-SPAN BENT.

Moments in Column C

K's of Columns B, F and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 145.

FIVE-SPAN BENT.

Moments in Column C

K's of Columns C, D and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 146.

FIVE-SPAN BENT.

Moments In Column C.

K's of Columns C, E, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 147.

FIVE-SPAN BENT.

Moments in Column C.

K's of Columns C, F and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 148.

FIVE-SPAN BENT.

Moments in Column C

K's of Columns D, E and One Other Column Change Simultaneously.

K's of All Other Columns Unity.

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary.

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Diagram 149.

FIVE-SPAN BENT.

Moments in Colum C.

K's of Columns D, F, and One Other Column Change Simultaneously.

K's of All Other Columns Unity.

K's of All Girders Unity.

Numbers on Curves indicate magnitudes of K in members which vary

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Diagram 150.
FIVE-SPAN BENT.
Moments in Column C.
K's of Columns D, F, and One Other Column Change Simultaneously.
K's of All Other Columns Unity.
K's of All Girders Unity.
Numbers on Curves indicate magnitudes of K in members which vary.
Moments in Terms of Wh
0.0
0.5
1.0
1.5
2.0
∞
Cols. E, F and A vary
Cols. E, F and B vary
Cols. E, F and C vary
Cols. E, F and D vary
Cols. E and F vary
Cols. E and F vary
0
.01
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